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W_2-RECURRENT LP-SASAKIAN MANIFOLD

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Abstract

In this paper, we study the properties of the W_2-recurrent LP-Sasakian manifold. We prove symmetric and skew-symmetric properties of the W_2-curvature tensor.

1. Introduction

In this paper, we shall study the W_2-recurrent LP-Sasakian manifold. An n-dimensional real differentiable manifold M_n is said to be *Lorentzian Para (LP)-Sasakian manifold* if it admits a (1, 1) tensor field F, a C^∞ vector field T, a C^∞ 1-form A and a Lorentzian metric g which satisfy (Mishra [2]):

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$$A(T) = -1, \tag{1.1}$$

$$\overline{\overline{X}} = X + A(X)T, \tag{1.2}$$

$$g(\overline{X}, \overline{Y}) = g(X, Y) + A(X)A(Y), \tag{1.3}$$

$$g(X, T) = A(X), \ D_X T = \overline{X}, \tag{1.4}$$

$$(D_X F)(Y) = \{g(X, Y) + A(X)A(Y)\}T + \{X + A(X)\}A(Y), \tag{1.5}$$

where $\overline{X} = F(X)$.

In an LP-Sasakian manifold M_n with structure (F, T, A, g), it can be seen that (Pokhariyal [5]):

$$\overline{T} = 0, \ A(X) = 0, \tag{1.6}$$

$$rank(F) = n - 1. \tag{1.7}$$

Moreover, if we put

$$'F(X, Y) = g(\overline{X}, Y), \tag{1.8}$$

then the tensor $'F(X, Y)$ is symmetric in X and Y.

In an n-dimensional LP-Sasakian manifold with the structure (F, T, A, g), we have the Riemannian curvature tensor as:

$$R(X, Y, Z, T) = g(Y, Z)A(X) - g(X, Z)A(Y), \tag{1.9}$$

where $g(X, Z)$ is the metric tensor representing potential and

$$Ric(X, Y) = g(QX, Y), \tag{1.10}$$

is the Ricci tensor representing the matter tensor. Pokhariyal and Mishra [3] have defined a tensor

$$'W_2(X, Y, Z, U)$$

$$= 'R(X, Y, Z, U) + \frac{1}{(n-1)}[g(X, Z)Ric(Y, U) - g(Y, Z)Ric(X, U)] \tag{1.11}$$

to study its physical and geometric properties. This tensor is skew-symmetric in X and Y. Breaking this tensor into skew-symmetric and symmetric parts, on contraction vanishes in an Einstein space. This allows Pirani formulation of gravitational waves to the Einstein space with the help of skew-symmetric part (Pokhariyal [7]). However, W_2 does not satisfy the cyclic property.

2. W_2-LP Sasakian Manifold

In this section, we study some of the geometrical properties of W_2-curvature tensor which is recurrent on LP-Sasakian manifold M_n in a Sasakian manifold.

If we consider an LP-Sasakian manifold M_n which is W_2-recurrent, then we have (Pokhariyal [4])

$$(D_U W_2)(X, Y)Z = B(U)W_2(X, Y)Z, \tag{2.1}$$

where B is a non-zero 1-form and $'W_2$ curvature tensor is given by

$$'W_2(X, Y)Z = R(X, Y)Z + \frac{1}{(n-1)}[g(X, Y)QY - g(Y, Z)QX]. \tag{2.2}$$

It is noted that Q is the symmetric endomorphism of the tangent space at each point to the Ricci tensor

$$QX = (n-1)X. \tag{2.3}$$

It is shown by De and Guha [1] that if in a Riemannian manifold (2.1) holds, then

$$R(X, Y)W_2(Z, U)V = 0, \tag{2.4}$$

where $R(X, Y)Z$ is simply the derivation of the tensor algebra at each point of M_n for its tangent vectors.

Using (1.4), we have

$$'W_2(X, Y, Z, T) = g(W_2(X, Y)Z, T) = A(W_2(X, Y)Z). \tag{2.5}$$