Lesson 4

- **Demonstration of Ohm’s Law**
  - Electromotive force (EMF), internal resistance and potential difference
  - Power and Energy

- **Applications of Ohm’s Law**
  - Resistors in Series and Parallel
  - Cells in series and Parallel
  - Kirchhoff’s Rules
  - Voltage Divider Circuit
  - Measuring Instruments
    - Potentiometer
    - Rayleigh Potentiometer
    - Wheatstone Bridge
    - Slide-Wire (Metre) Bridge

- **RC Circuits**
- **Electrolysis**
EMF, Internal Resistance and Potential Difference

- The emf of a battery is the maximum possible voltage that the battery can provide between its terminals.

- The e.m.f of this battery is given as

\[ \varepsilon = I(R + r) \]
\[ \varepsilon = Ir + IR \]
\[ \varepsilon = Ir + V_{ab} \]

\[ V_{ab} = V_b - V_a = \text{terminal voltage (potential difference across the battery terminals)} \]

- \( R \) = external resistance
- \( r \) = internal resistance

\[ V_{ab} = V_{dc} \]
Power and Energy

- The electrical (potential) energy, \( W \) is the energy gained by the charge \( Q \) from a voltage source (battery) having a terminal voltage \( V \).

- \( W = QV \) (the work done by the source on the charge)

- But \( Q = It \), then \( W = Vit \) Unit: Joule (J)

- The rate of energy delivered to the external circuit by the battery is called the electric power given by,

- Unit: watt (1 W = 1J/s)

\[
P = VI \quad \text{but} \quad V = IR
\]

\[
P = I^2R \quad \text{or} \quad P = \frac{V^2}{R}
\]
EMF, Power and Energy

Example: Terminal Voltage of a Battery

A battery has an emf of 12.0 V and an internal resistance of 0.05 Ω. Its terminals are connected to a load resistance of 3.00 Ω.

(i) Find the current in the circuit and the terminal voltage of the battery.
(ii) Calculate the power delivered to the load resistor, the power delivered to the internal resistance of the battery, and the power delivered by the battery.

Solution

(i) \[ I = \frac{\mathcal{E}}{R + r} = \frac{12.0 \text{ V}}{3.05 \Omega} = 3.93 \text{ A} \]

\[ \Delta V = \mathcal{E} - Ir = 12.0 \text{ V} - (3.93 \text{ A})(0.05 \Omega) = 11.8 \text{ V} \]

\[ \Delta V = IR = (3.93 \text{ A})(3.00 \Omega) = 11.8 \text{ V} \]

(ii) \[ \mathcal{P}_R = I^2R = (3.93 \text{ A})^2 (3.00 \Omega) = 46.3 \text{ W} \]

\[ \mathcal{P}_r = I^2r = (3.93 \text{ A})^2 (0.05 \Omega) = 0.772 \text{ W} \]
Cells in series and Parallel

1. Current drawn from the cell:
   \[ = \frac{\text{cell emf}}{\text{total circuit resistance}} \]

2. PD across resistors in SERIES with the cell:
   \[ = \text{cell current} \times \text{resistance of each resistor} \]

3. Current through parallel resistors:
   \[ = \text{pd across the parallel resistors} \]
   \[ = \text{resistance of each resistor} \]
Cells in series and Parallel

TOTAL EMF
Case ‘a’ - Cells connected in the same direction
Add emfs together
In case ‘a’ total emf = 3.5V

Case ‘b’ - Cells connected in different directions
Total emf equals sum of emfs in one direction minus the sum of the emfs in the other direction
In case ‘b’ total emf = 0.5V in the direction of the 2V cell

TOTAL INTERNAL RESISTANCE
In both cases this equals the sum of the internal resistances
Cells in series and Parallel

In the circuit shown below calculate the current flowing and the pd across the 8 ohm resistor

Both cells are connected in the same direction.
Therefore total emf = 1.5 + 6.0
= 7.5 V

All three resistors are in series.
Therefore total resistance
= 4.0 + 3.0 + 8.0
= 15 Ω
Current = \( I = \frac{\varepsilon_T}{R_T} \)
= 7.5 / 15
current = 0.5 A

PD across the 8 ohm resistor
= \( V_8 = I \times R_8 \)
= 0.5 x 8
pd = 4 V
Cells in series and Parallel

For $N$ identical cells each of emf $\varepsilon$ and internal resistance $r$

Total emf = $\varepsilon$

Total internal resistance = $r / N$

The lost volts = $I r / N$ and so cells placed in parallel can deliver more current for the same lost volts due to the reduction in internal resistance.
Resistors in Series and Parallel

for a series combination of two resistors, the currents are the same in both resistors because the amount of charge that passes through $R_1$ must also pass through $R_2$ in the same time interval.

$$I_1 = I_2 = I$$

$$R_{eq} = R_1 + R_2$$

$$\Delta V = IR_1 + IR_2 = I(R_1 + R_2)$$

when resistors are connected in parallel, the potential differences across the resistors is the same.

$$I = I_1 + I_2 = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} = \Delta V \left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{\Delta V}{R_{eq}}$$
Resistors in Series and Parallel

The inverse of the equivalent resistance of two or more resistors connected in parallel is equal to the sum of the inverses of the individual resistances. Furthermore, the equivalent resistance is always less than the smallest resistance in the group.
Resistors in Series and Parallel

Calculate the potential difference across and the current through the 6 ohm resistor in the circuit below.

Total resistance of the circuit
= 8 Ω in series with 12 Ω in parallel with 6 Ω
= 8 + 5.333
= **13.333 Ω**

Total current drawn from the battery
= \( V / R_T \)
= 9V / 13.333 Ω
= **0.675 A**

pd across 8 Ω resistor = \( V_8 = I \times R_8 \)
= 0.675 A x 8 Ω
= **5.40 V**

therefore pd across 6 Ω (and 12 Ω) resistor, \( V_6 \)
= 9 – 5.4

pd across 6 Ω resistor = **3.6 V**

Current through 6 Ω resistor = \( I_6 = V_6 / R_6 \)
= 3.6 V / 6 Ω
**current through 6 Ω resistor = 0.600 A**
Kirchhoff’s Rules

• There are ways in which resistors can be connected so that the circuits formed cannot be reduced to a single equivalent resistor

• Two rules, called Kirchhoff’s Rules can be used instead

• (i) Junction Rule
  – The sum of the currents entering any junction must equal the sum of the currents leaving that junction
    • A statement of Conservation of Charge
      \[ \Sigma I = 0, \]

• (ii) Loop Rule
  – The sum of the potential differences across all the elements around any closed circuit loop must be zero
    • A statement of Conservation of Energy
      \[ \Sigma R I = \Sigma E. \]
Kirchhoff’s Rules

• $I_1 = I_2 + I_3$
• From Conservation of Charge
• Diagram b shows a mechanical analog
Kirchhoff’s Rules

• Assign symbols and directions to the currents in all branches of the circuit
  – If a direction is chosen incorrectly, the resulting answer will be negative, but the magnitude will be correct

• When applying the loop rule, choose a direction for transversing the loop
  – Record voltage drops and rises as they occur
Kirchhoff’s Rules

• Traveling around the loop from a to b
  • In a, the resistor is transversed in the direction of the current, the potential across the resistor is $-IR$
  • In b, the resistor is transversed in the direction opposite of the current, the potential across the resistor is $+IR$
Kirchhoff’s Rules

- In c, the source of emf is transversed in the direction of the emf (from – to +), the change in the electric potential is +ε.
- In d, the source of emf is transversed in the direction opposite of the emf (from + to -), the change in the electric potential is -ε.
Kirchhoff’s Rules

• Use the junction rule as often as needed, so long as, each time you write an equation, you include in it a current that has not been used in a previous junction rule equation
  – In general, the number of times the junction rule can be used is one fewer than the number of junction points in the circuit
Kirchhoff’s Rules

• The loop rule can be used as often as needed so long as a new circuit element (resistor or battery) or a new current appears in each new equation

• You need as many independent equations as you have unknowns
Voltage Divider Circuit

Identify the current and Apply KVL

\[ v_S = iR_1 + iR_2 = i(R_1 + R_2) \]

\[ i = \frac{v_S}{R_1 + R_2} \]

\[ v_1 = iR_1 = v_S \frac{R_1}{R_1 + R_2} \]

\[ v_2 = iR_2 = v_S \frac{R_2}{R_1 + R_2} \]

\[ v_O = \frac{R_2}{R_1 \left[ 1 + \left( \frac{R_2}{R_L} \right) \right] + R_2} v_S \]

\[ R_L \to \infty, \]

\[ v_O \to \frac{R_2}{R_1 + R_2} \]
Potentiometer

• A potentiometer is mainly used to measure potential difference.
• It consists of a uniform wire.
• Basically a potentiometer circuit consists of a uniform wire AB of length 100.0cm, connected in series to a driver cell with emf $V$ of negligible internal resistance.
• Potentiometer can be used to:
  i) Measure an unknown e.m.f. of a cell.
  ii) Compare the e.m.f.s of two cells.
  iii) Measure the internal resistance of a cell.

All the uses of the potentiometer depend on the fact that it can measure potential difference accurately, and without drawing current from the circuit under test.
The potentiometer is balanced when the jockey (sliding contact) is at such a position on wire AB that there is no current through the galvanometer. Thus:

- **Galvanometer reading = 0**

When the potentiometer in balanced, the unknown voltage (potential difference being measured) is equal to the voltage across AC.

\[ V_{AC} = \varepsilon = \left( \frac{V}{R_{AB}} \right) \times \left( \frac{l_{AC}}{l_{AB}} R_{AB} \right) \]

\[ \varepsilon = \frac{l_{AC}}{l_{AB}} V \]

\[ V_x = V_{AC} \]
When the potentiometer is balanced, 
\[ I_G = 0 \]

Balance length,
\[ l_{AC} = l_1 \text{ for } \varepsilon_1 \text{ and } l_{AD} = l_2 \text{ for } \varepsilon_2 \]

\[ \varepsilon_1 = \frac{l_{AC}}{l_{AB}} V = \frac{l_1}{l_{AB}} V \]
\[ \varepsilon_2 = \frac{l_{AD}}{l_{AB}} V = \frac{l_2}{l_{AB}} V \]

\[ V = \frac{l_{AB}}{l_1} \varepsilon_1 = \frac{l_{AB}}{l_2} \varepsilon_2 \]

\[ \therefore \frac{l_2}{l_1} = \frac{\varepsilon_2}{\varepsilon_1} \]

Hence
\[ \frac{l_2}{l_1} = \frac{\varepsilon_2}{\varepsilon_1} \]
Rayleigh Potentiometer

It consists of two plug-type resistance boxes, $R_1$, $R_2$, joined in series. At the start of a measurement all the plugs of $R_1$ are inserted, and all of $R_2$ taken out. Then $R_1$ is zero, and the main current $I$ sets up no potential difference across it; but when the key $K$ is pressed, the unknown e.m.f. $E$ deflects the galvanometer. $R_1$ is now increased by, say, 100 ohms, and $R_2$ is
Rayleigh Potentiometer

decreased by the same amount. In this way $R_1 + R_2$ is kept constant, and the current $I$ does not change. But there is now a potential difference across $R_1$, which opposes $E$. Plugs are taken out of $R_1$ and put into $R_2$, so as to keep $R_1 + R_2$ constant, until the galvanometer shows no deflection when $K$ is pressed. If $R'_1$ is the value of $R_1$ at this point, then

$$E = R'_1 I.$$ 

The procedure is now repeated with a standard cell of e.m.f. $E_0$, in place of $E$. Since $R_1 + R_2$ has been kept constant, the current $I$ is the same as before; hence, if $R''_1$ is the new value of $R_1$ at balance,

$$E_0 = R''_1 I.$$ 

Consequently,

$$\frac{E}{E_0} = \frac{R'_1 I}{R''_1 I} = \frac{R'_1}{R''_1}.$$
Wheatstone Bridge

• It is used to measure the unknown resistance of the resistor.
• Figure below shows the Wheatstone bridge circuit consists of a cell of e.m.f. \( \varepsilon \) (accumulator), a galvanometer, known resistances \((R_1, R_2\) and \(R_3)\) and unknown resistance \(R_x\).
• The Wheatstone bridge is said to be balanced when no current flows through the galvanometer this can be achieved by adjusting \(R_3\). Hence

\[
I_1R_1 = I_2R_3
\]

\[
I_1R_2 = I_2R_x
\]

Potential at C = Potential at D

\[
R_x = \left(\frac{R_2}{R_1}\right)R_3
\]
Wheatstone Bridge

Alternative Wheatstone Bridge proof

\[ V_{AB} = V_{CB}, \quad \frac{V_{AB}}{V_{AD}} = \frac{V_{CB}}{V_{CD}}. \]

Also, since \( I_g = 0 \), \( P \) and \( R \) carry the same current, \( I_1 \), and \( X \) and \( Q \) carry the same current, \( I_2 \). Therefore

\[ \frac{V_{AB}}{V_{AD}} = \frac{I_1 P}{I_1 R} = \frac{P}{R}, \]
\[ \frac{V_{CB}}{V_{CD}} = \frac{I_2 Q_2}{I_2 X} = \frac{Q}{X}. \]

Hence

\[ \frac{P}{R} = \frac{Q}{X}, \]
\[ \frac{P}{Q} = \frac{R}{X}. \]
Slide-Wire (Metre) Bridge

\[
\frac{X}{R} = \frac{R_{AC}}{R_{CB}}.
\]

\[
\frac{X}{R} = \frac{l_1}{l_2}.
\]
RC Circuits

- Charging a Capacitor

\[ \mathcal{E} - \frac{q}{C} - IR = 0 \]

\[ I_0 = \frac{\mathcal{E}}{R} \quad Q = C\mathcal{E} \quad \frac{dq}{dt} = \frac{\mathcal{E}}{R} - \frac{q}{RC} \]

\[ i = \frac{dq}{dt} = C \frac{dv}{dt} \]
RC Circuits

• Charging a Capacitor

To find an expression for $q$, we solve this separable differential equation. We first combine the terms on the right-hand side:

$$\frac{dq}{dt} = \frac{CE}{RC} - \frac{q}{RC} = -\frac{q - CE}{RC}$$

Now we multiply by $dt$ and divide by $q - CE$ to obtain

$$\frac{dq}{q - CE} = -\frac{1}{RC} \ dt$$

Integrating this expression, using the fact that $q = 0$ at $t = 0$, we obtain

$$\int_{0}^{q} \frac{dq}{q - CE} = -\frac{1}{RC} \int_{0}^{t} dt$$

$$\ln \left( \frac{q - CE}{CE} \right) = -\frac{t}{RC}$$

$$q(t) = CE (1 - e^{-t/RC}) = Q(1 - e^{-t/RC})$$

$$I(t) = \frac{E}{R} e^{-t/RC}$$
RC Circuits

• Charging a Capacitor

Plots of capacitor charge and circuit current versus time are shown in the Figure. Note that the charge is zero at $t = 0$ and approaches the maximum value $CE$ as $t \to \infty$. The current has its maximum value $I_0 = \varepsilon/R$ at $t = 0$ and decays exponentially to zero as $t \to \infty$. The quantity $RC$, which appears in the exponents, is called the time constant $\tau$ of the circuit. It represents the time interval during which the current decreases to $1/e$ of its initial value; that is, in a time interval $\tau$, $I = e^{-1}I_0 = 0.368I_0$. In a time interval $2\tau$, $I = e^{-2}I_0 = 0.135I_0$, and so forth. Likewise, in a time interval $\tau$, the charge increases from zero to $CE[1 - e^{-1}] = 0.632CE$. 
RC Circuits

• Discharging a Capacitor

\[- \frac{q}{C} - IR = 0\]

When we substitute \( I = \frac{dq}{dt} \) into this expression, it becomes

\[-R \frac{dq}{dt} = \frac{q}{C}\]

\[\frac{dq}{q} = -\frac{1}{RC} \, dt\]

Integrating this expression, using the fact that \( q = Q \) at \( t = 0 \) gives

\[\int_{Q}^{q} \frac{dq}{q} = -\frac{1}{RC} \int_{0}^{t} dt\]

\[\ln \left( \frac{q}{Q} \right) = -\frac{t}{RC}\]

\[q(t) = Qe^{-t/RC}\]

\[I(t) = \frac{dq}{dt} = \frac{d}{dt} (Qe^{-t/RC}) = -\frac{Q}{RC} e^{-t/RC}\]
An uncharged capacitor and a resistor are connected in series to a battery. If $\varepsilon = 12.0 \text{ V}$, $C = 5.00 \mu\text{F}$, and $R = 8.00 \times 10^5 \Omega$, find the time constant of the circuit, the maximum charge on the capacitor, the maximum current in the circuit, and the charge and current as functions of time.

**Example: Charging a Capacitor in an RC Circuit**

- The time constant of the circuit is $\tau = RC = (8.00 \times 10^5 \Omega)(5.00 \times 10^{-6} \text{ F}) = 4.00 \text{ s}$. The maximum charge on the capacitor is $Q = C\varepsilon = (5.00 \mu\text{F})(12.0 \text{ V}) = 60.0 \mu\text{C}$. The maximum current in the circuit is $I_0 = \frac{\varepsilon}{R} = \frac{12.0 \text{ V}}{8.00 \times 10^5 \Omega} = 15.0 \mu\text{A}$. Using these values, we find that:

$$q(t) = (60.0 \mu\text{C})(1 - e^{-t/4.00 \text{ s}})$$

$$I(t) = (15.0 \mu\text{A})e^{-t/4.00 \text{ s}}$$
RC Circuits

- Consider a capacitor of capacitance $C$ that is being discharged through a resistor of resistance $R$, as shown in the Figure below.

(i) After how many time constants is the charge on the capacitor one-fourth its initial value?

(ii) The energy stored in the capacitor decreases with time as the capacitor discharges. After how many time constants is this stored energy one-fourth its initial value?

Solution

(i) 
\[
\frac{Q}{4} = Q e^{-t/RC}
\]
\[
\frac{1}{4} = e^{-t/RC}
\]
Taking logarithms of both sides, we find
\[
-\ln 4 = -\frac{t}{RC}
\]
\[
t = RC \ln 4 = 1.39RC = 1.39\tau
\]

(ii) 
\[
U = \frac{q^2}{2C} = \frac{Q^2}{2C} e^{-2t/RC} = U_0 e^{-2t/RC}
\]
where $U_0 = Q^2/2C$ is the initial energy stored in the capacitor. As in part (A), we now set $U = U_0/4$ and solve for $t$:
\[
\frac{U_0}{4} = U_0 e^{-2t/RC}
\]
\[
\frac{1}{4} = e^{-2t/RC}
\]
Again, taking logarithms of both sides and solving for $t$ gives
\[
t = \frac{1}{2} RC \ln 4 = 0.693RC = 0.693\tau
\]
Electrolysis

- The chemical Effect of Current

**Electrolysis** is the breakdown of a substance by electricity

**Electrolyte** - a molten or aqueous solution through which an electrical current can flow.

- Electrolytes contain positive and negative ions.
- During electrolysis, positive and negative electrodes are put into the electrolyte.
- The positive electrode is called the anode.
- The negative electrode is called the cathode.
- The negative ions (called anions) are attracted to the anode.
- At the anode, the negative ions lose electrons to become atoms/molecules.
- The positive ions (called cations) are attracted to the cathode.
- At the cathode, the positive ions gain electrons to become atoms/molecules.
The whole arrangement is called a *voltameter*, presumably because it can be used to measure the current delivered by a voltaic cell; if the electrolyte is a solution of a copper or silver salt, the voltameter is called a copper or silver voltameter. If the electrolyte is acidulated water, then the voltameter is called a water voltameter, because when a current passes through it, the water, not the acid, is decomposed.
Electrolysis

Faraday’s Laws of Electrolysis

The mass of any substance liberated in electrolysis is proportional to the quantity of electric charge that liberated it.

The mass of a substance which is liberated by one coulomb is called its electrochemical equivalent. It is expressed in kilogrammes per coulomb (kg C$^{-1}$) in SI units. If $z$ is the electrochemical equivalent of a substance, the mass of it in kilogrammes liberated by $I$ amperes in $t$ seconds is

$$m = zIt.$$ 

$$I = \frac{m}{zt}.$$