APH 101: GENERAL PHYSICS (Jan-May 2015)

LECTURER: DR. Z. BIRECH

COURSE CONTENT

1. **MECHANICS:** Units, Dimensional analysis, Scalars and Vectors, Kinematics and Newton's laws of motion, Work, Energy, Power, Collisions and Momentum, Fluid Mechanics, Fluids at rest, Oscillations and Waves.
2. **HEAT AND THERMODYNAMICS:** Measurement of Temperature, Measurement of Heat, Newton's law of cooling, Latent Heat, Expansion of Solids, Liquids and Gases, Gas Laws, Change of State, Modes of Heat transfer.
3. **ELECTRICITY & MAGNETISM:** Electric charges, Electric field and Gauss' Law, Electric potential, Capacitors, Current, Resistance, Electric Circuits, Magnetic Field, Ampere's Law, Faraday's Law, Elementary electronics.
4. **MODERN PHYSICS**: The Nucleus, Isotopes, Nuclear Reactions, Radioactive Decay.
5. **OPTICS:** Light as an Electromagnetic Wave, Propagation of Light, Reflection and Refraction of plane waves, Interference.

REFERENCES

1. Fundamental of Physics by Halliday/Resnick/Walker
2. University Physics with Modern Physics by Young and Freedman
3. Principles of Physics by Frank J B
4. Introduction to Physics for Scientists and Engineers by Fredrick Bueche
5. Advanced Level Physics by Nelkon and Parker

MECHANICS AND PROPERTIES OF MATTER

UNITS

To describe and characterize physical phenomena, scientists and engineers must agree on a consistent set of units with which measurements are to be compared. The unit is simply the standard yardstick with which a particular event is contrasted. For example quantities such as mass, time, currents, velocities and others, become meaningful only in comparison with the ones familiar to us. Unfortunately, in the historical development of science, different systems of units were used in different parts of the world and in the same country by different professions. As a result, several diverse units have been in common use. These are now being replaced under international agreement, by the 'International System of Units' or SI units. In this system, the **metre, kilogram** and **second** are the **fundamental units** of **length, mass** and **time** respectively. The SI units are now widely used throughout the world and, as a result, the use of the metric (as is called) system of units makes the physics, and by extension the whole of science, more lucid and the computation more tractable. Although we shall often use SI units, it is **important to know how to convert from one unit to another**. Some of the conversion factors are listed below:

LENGTH

1 inch = 2.54 cm

1 foot = 12 inches = 30.48 cm

1 metre = 1.0936 yards = 3.281 feet = 39.37 inches

1 mile = 5280 feet = 1760 yards = 1.609 kilometres

VOLUME

1 litre = $1000 cm^3 = 10^{-3} m^3$

$= 3.531 \times 10^{-2}$ ft^3

1 gallon = 3.786 litres

MASS

1 kg = 10^3 gm

1 kg = 6.022×10^{26} u

1 u = 1.66×10^{-27} kg

TIME

1 hr = 60min = 3600 s

1 day = 24 hrs = 8.64×10^4s

FORCE

1 pound = 4.448 Newton

1 Newton = 0.2248 pounds

PRESSURE

1 atmosphere = 1.01325 bar

$= 1.01325 \times 10^5$Pascal

= 760 torr

ENERGY

1 Calorie = 4.184 Joules

1 electron volt = 1.602$\times$10-19 Joules

POWER

1 horsepower = 745.7 Watts

ANGLES

π Radians = 180^o

1 radian = 57.30^o

1^o = 1.745$\times$10-2 radians

Example: A dairy-transporting van is driving at a speed of 50 miles per hour. Determine the speed of the van in kilometres per hour and in metres per second?

Solution: 1 mile $\approx 1.61km$. Denoting the speed of the van by *v*, we have

$$v = \left(\frac{50mi}{1hr}\right)\left(\frac{1.61km}{1mi}\right) = 80.5kmhr^{-1}$$

To convert to metres per second, we have *1hr = 3600* seconds and *1 km = 1000m*. Hence,

$$v = 80.5kmhr^{-1} = \left(\frac{80.5km}{1hr}\right)\left(\frac{1000m}{1km}\right)\left(\frac{1}{\frac{3600s}{1hr}}\right) = 22.4ms^{-1}$$

To express the very large and very small quantities that we often encounter in science and engineering, we use the **scientific notation** which **employs the powers of ten** (10). In this notation, for example, 3560000000*m* and 0.000000492*s* can be written as $3.56\times10^{9}m$ and $4.92\times10^{-7}s$ respectively.

DERIVED UNITS AND DIMENSIONAL ANALYSIS

Quantities that concern scientists are not limited to mass, length and time only. We often describe the behaviour of objects in terms of their velocities, we need to identify the nature of forces that act on bodies, we pay for the energy consumed by appliances and are curious about the power a motor can deliver, atmospheric pressure is a useful indicator of weather conditions etc. All these apparently disparate properties measured in the **units** metres per second (velocity), Newton (force), joule (energy), watt (power) and Pascal (pressure) are ultimately **expressible as products of powers of mass, length and time**. **These units** are therefore **known** as **derived units**, to distinguish them from the three fundamental units (kilogram, metre, second).

The **numerical specification** of a particular **quantity** such as **distance** or speed, for example, **depends** on the **system of units** we employ. For instance, cars travelling at 50*mph*, 80.5 $kmhr^{-1}$ and 22.4 ms^{-1} are all going at the same speed. **But** note that the **combination of units** used to **characterize speed** is always the same, namely the ratio of **Length/Time**. The **type of unit involved** is called the **dimension** of the variable **and does not depend on the system of units** that is used. We use square brackets [ ] to indicate the dimension of the variable. For example, the dimension of speed is [L]/[T], that of distance is [L]and that of volume is $[L]^3$. Thus when we say that a car gets 30 miles per gallon, the dimension of that variable is

$$[L]/[L]^3 = [L]^{-2}$$

Equations that relate various physical quantities must be dimensionally homogenous. By this we mean that if an equation reads that

$$A = B + C$$

then the terms *A*, *B* and *C* must all have the same dimensions. To be specific, we cannot compare apples and oranges.

Example: Show that the equation for impulse $\vec{F}t = m\vec{v} - m\vec{u}$ is dimensionally correct.

Solution: From the equation $\vec{F}t = m\vec{v} - m\vec{u}$, the units involved on both sides of the equation are

$$Ns = kg\ m\ s^{-1}$$

$$kg\ ms^{-2}\ s = kg\ m\ s^{-1}$$

$$kg\ m\ s^{-1} = kg\ m\ s^{-1}$$

$$\frac{[M][L]}{[T]} = \frac{[M][L]}{[T]}$$ or $[M][L][T]^{-1}$, hence correct.

SCALARS AND VECTORS

Physical quantities can be classified into two main categories, namely scalars and vectors. A scalar is a quantity that can be specified by its magnitude (size) together with appropriate units. Examples include Temperature (^oC or *K*), Density (kgm^{-3}), Energy (*J*), Speed (ms^{-1}), Distance (*m*), Mass (*kg*) etc. On the other hand, a vector is a quantity that is completely specified by its magnitude and direction together with the appropriate units. Examples include Velocity (ms^{-1}), Displacement (*m*), Acceleration (ms^{-2}), Force (*N*), Linear momentum ($kgms^{-1}$), Angular momentum (kgm^2s^{-1}) etc.

ADDITION OF VECTORS

There are two ways by which vectors can be combined or added. These are; graphical and vector component methods.

Graphical Method: Two or more vectors can be combined or added to find the vector sum also known as the resultant. This can be done by using either the triangle or parallelogram method.

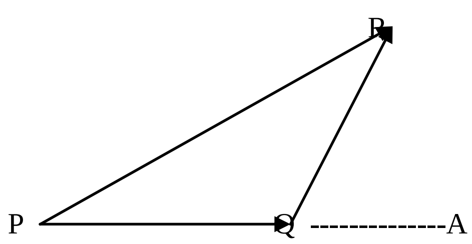


Figure 1: Triangle Method

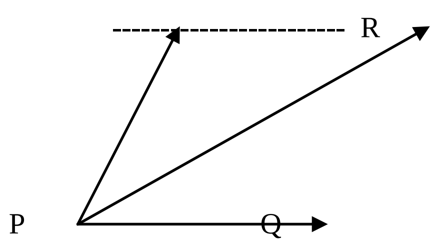


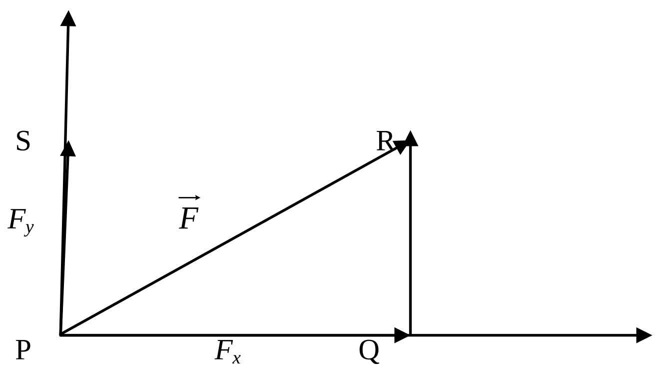
Figure 2: Parallelogram Method

From the first diagram, angle $Q\hat{P}R = \alpha$ and $A\hat{Q}R = \phi$. Since vectors in the same direction and with the same magnitude are equal, the methods are equivalent for it does not make a difference whether the line representing $\vec{T}_2$ is drawn from the tail or the head of the line representing $\vec{T}_1$.

Example 1: Suppose the magnitude of $\vec{T}_1$ is 140N and that of $\vec{T}_2$ is 110N with the angle ϕ equal to 30°. Determine the magnitude of the resultant $\overrightarrow{PR}$. **Hint**: Choose a convenient scale, say 1*cm* to represent 20N. Complete the construction and measure *PR* and angle $Q\hat{P}R$.

Solution: PR = 12*cm* = 240N and $Q\hat{P}R$ = 13°

Vector Components: Although any number of vectors can be added in the manner described above, the process is slow and tedious. Furthermore, the graphical method is not very accurate. A quick and more convenient method makes use of vector components. Breaking up a vector into its components is known as resolving a vector. Using trigonometry, a vector can be resolved into its components as in the figure below:



From the figure, if angle $Q\hat{P}R$ is β, then $F_x = F\cos\beta$ and $F_y = F\sin\beta$, where F is the magnitude of $\vec{F}$. Hence, the right-angled triangle *PQR* and the fact that *QR* also represents F_y gives;

$$F^2 = (PQ)^2 + (QR)^2 = F_x^{\ 2} + F_y^{\ 2}$$

or

$$F = \sqrt{F_x^{\ 2} + F_y^{\ 2}}$$

and the angle the resultant makes with the horizontal is given by

$$\tan\beta = \frac{F_y}{F_x}$$

or

$$\beta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

Example 2: Solve example 1 above using the method of components given that $\vec{T}_1$ is 140N, $\vec{T}_2$ is 110N and the angle ϕ is 30°. Find α and $\vec{T}$.

Solution:

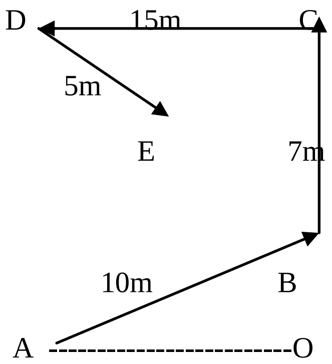
Vector	***x*-component**	***y*-component**
T_1 = 140N	T_1 = 140N	0
T_2 = 110N	T_2cos30 = 95.26N	T_2sin30 = 55.0N
Total/Sum	**235.26**	**55.0**

The magnitude of the resultant vector is then given as

$$T = \sqrt{T_x^{\ 2} + T_y^{\ 2}} = \sqrt{(235.26)^2 + (55)^2} = 241.6N$$

Note: The analytical results are in close agreement with those obtained graphically, but the analytical method is quick and more accurate.

A component which points in the negative *x* or *y* or *z* direction must be considered negative since it represents a displacement in the –*x* or –*y* or –*z* direction. **For example**, consider the figure below. If angle $O\hat{A}B$ is 30° and $E\hat{D}C$ is 37°, find the resultant displacement.



Solution: We resolve each vector as follows:

Vector	***x*-component**	***y*-component**
10*m*	10cos30 = 8.7*m*	10sin30 = 5.0*m*
7*m*	0*m*	7*m*
15*m*	-15*m*	0*m*
5*m*	5cos63 = 4.0*m*	-5sin63 = -3.0*m*
Total/Sum	**-2.3m**	**9.0m**

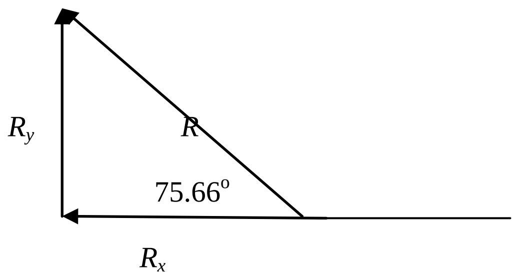
Therefore, the resultant vector R is

$$R = \sqrt{(-2.3m)^2 + (9.0m)^2} = 9.3m$$

The angle the resultant vector makes with the horizontal axis is

$$\tan\phi = \frac{9}{-2.3} = -3.913 \text{ or } \phi = -75.66^o$$

This can be represented in diagram form as shown below.



UNIT VECTORS

A unit vector is a vector with magnitude of exactly one (1) and only points in a particular direction. It lacks both dimension and units and its sole purpose is to point or specify a direction. Unit vectors in the positive directions of the *x*, *y* and *z* axes are labelled $\hat{i}$, $\hat{j}$ and $\hat{k}$ respectively. The unit vectors are very useful for expressing other vectors. For example, we can express vectors $\vec{a}$ and $\vec{b}$ as

$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

$$\vec{b} = b_x \hat{i} + b_y \hat{j} + b_z \hat{k}$$

The quantities $a_x \hat{i}$, $a_y \hat{j}$ and $a_z \hat{k}$ are vectors and are called the vector components of $\vec{a}$. The quantities a_x, a_y and a_z are scalars and are known as the scalar components of $\vec{a}$ or simply its components. For example, if

$$\vec{a} = 10\hat{i} - 6\hat{j} + 2\hat{k}$$

$$\vec{b} = -4\hat{i} + 3\hat{j} - 7\hat{k}$$

then, 10, 6 and 2 (4, 3 and 7) are the scalar components of the vector $\vec{a}$ ($\vec{b}$) in the positive (negative) $\hat{i}$, negative (positive) $\hat{j}$ and positive (negative) $\hat{k}$ directions respectively. In general, a vector $\vec{F}$ with components along *x*, *y* and *z* axes is given by

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

Since the unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$ are mutually orthogonal, their scalar products satisfy the following properties:

$$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$$

$$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$$

$$\hat{i}\times\hat{j} = \hat{k}, \quad \hat{j}\times\hat{k} = \hat{i} \text{ and } \hat{k}\times\hat{i} = \hat{j}$$

$$\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$$

Also, if $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

then $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$

and $\vec{A}+\vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$

VECTOR MULTIPLICATION

Scalar Product (Dot Product)

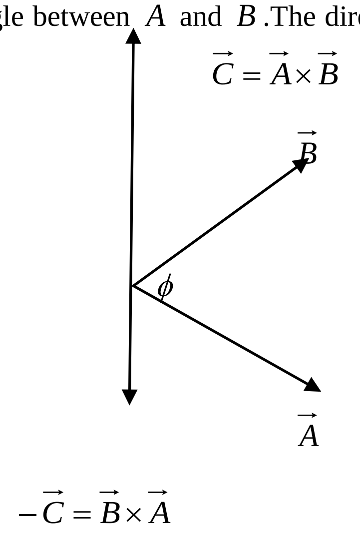
The scalar product of two vectors is defined as

$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\phi$$

Vector Product (Cross Product). If the vector $\vec{C} = \vec{A}\times\vec{B}$, then the magnitude of $|\vec{C}|$ is defined by

$$|\vec{C}| = |\vec{A}\times\vec{B}| = |\vec{A}||\vec{B}|\sin\phi$$

where ϕ is the angle between $\vec{A}$ and $\vec{B}$.The direction of $\vec{C}$ is perpendicular to $\vec{A}$ and $\vec{B}$ as in the figure below:



Example: Two vectors $\vec{A}$ and $\vec{B}$ are given by $\vec{A} = 3\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{B} = -\hat{i} + 2\hat{j} + 6\hat{k}$. Calculate

(a) The magnitudes of $\vec{A}$ and $\vec{B}$

(b) The scalar product $\vec{A} \cdot \vec{B}$

(c) The angle between the two vectors.

Solution: (a) Magnitude of *A* is

$$\left|\vec{A}\right| = \sqrt{A^2} = \sqrt{\left|\vec{A}\right| \cdot \left|\vec{A}\right|} = \sqrt{\left(3\hat{i} + 4\hat{j} - 5\hat{k}\right) \cdot \left(3\hat{i} + 4\hat{j} - 5\hat{k}\right)} = \sqrt{50}$$

Magnitude of *B* is

$$\left|\vec{B}\right| = \sqrt{B^2} = \sqrt{\left|\vec{B}\right| \cdot \left|\vec{B}\right|} = \sqrt{\left(-\hat{i} + 2\hat{j} + 6\hat{k}\right) \cdot \left(-\hat{i} + 2\hat{j} + 6\hat{k}\right)} = \sqrt{41}$$

(b) The scalar product is $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$. Given that $A_x = 3, A_y = 4, A_z = -5$ and

$B_x = -1$, $B_y = 2$, $B_z = 6$, then

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$= -3 + 8 - 30$$

$$= -25$$

(c) From the definition of scalar product, we have

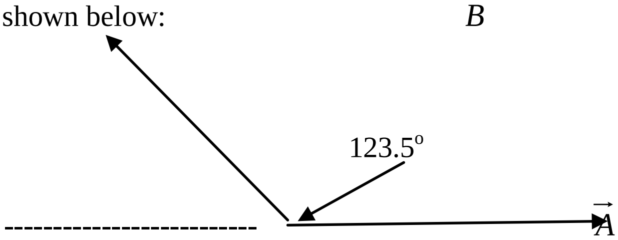
$$\vec{A} \cdot \vec{B} = \left|\vec{A}\right|\left|\vec{B}\right| \cos\phi$$

so that

$$\cos\phi = \frac{\vec{A} \cdot \vec{B}}{\left|\vec{A}\right| \cdot \left|\vec{B}\right|} = \frac{-25}{\sqrt{50}\sqrt{41}} = -0.5522$$

Hence, $\phi = \cos^{-1}(-0.5522) = 123.5^o$

This can be sketched as shown below:



MOTION IN ONE AND TWO DIMENSIONS

The brief mathematical concept of vectors developed above is very useful for the description of displacement, velocity and acceleration in one, two or three dimensions. This is because a body can undergo either one of the following types of motion or a combination of two or more of these motions:

i **Translational** or rectilinear motion i.e. motion in a straight line

ii **Rotational** or circular motion e.g. a rotating wheel or planets around the sun

iii **Vibrational** or oscillatory motion e.g. a pendulum clock, atoms or electrons in a metal or solid.

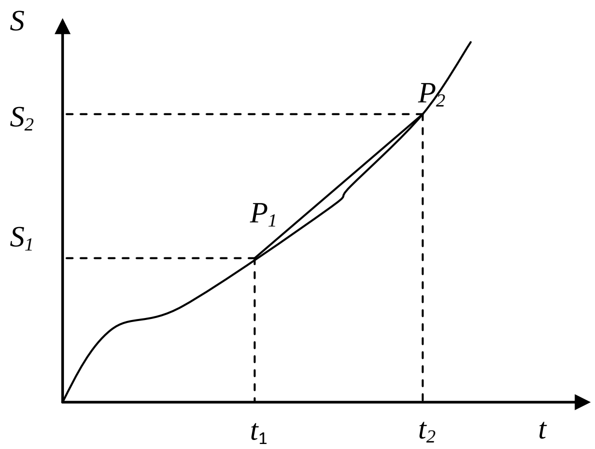
VARIOUS PHYSICAL QUANTITIES ASSOCIATED WITH MOTION

Displacement $(\vec{S})$: is a physical quantity that specifies the position of an object relative to the initial point (origin) or it is the distance moved in a given direction.

Velocity $(\vec{v})$**:** is vector that specifies the rate of change of displacement with time. It is defined by

$$\bar{v} = \frac{Change\ in\ Displacement}{Time\,Taken} \quad \text{or} \quad \vec{v} = \frac{\vec{S}}{t} \qquad (1)$$

For example, consider the displacement-time graph of, say, an automobile given below:



If $S_2 - S_1$ is the change occurring in position in time interval $t_2 - t_1$, then the average velocity is

$$\bar{v} = \frac{S_2 - S_1}{t_2 - t_1} = \frac{\Delta\vec{S}}{\Delta t} = \text{slope of the straight line } P_1P_2.$$

On the other hand, the speed (v) between t_1 and t_2 is defined by

$$v = \frac{Actual\ Distance}{t_2 - t_1} = \frac{Curved\ Distance\ P_1P_2}{\Delta t} = \left|\vec{v}\right| = \left|\frac{\Delta \vec{S}}{\Delta t}\right| \qquad (2)$$

i.e. **speed** is the rate of change of distance with time (and in some cases is equal to the magnitude of the velocity).

Acceleration $(\vec{a})$ is a vector specifying how fast the velocity of a body changes with time i.e.

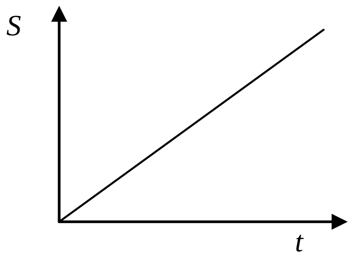
$$\vec{a} = \frac{Change\ in\ Velocity}{Time\ taken} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}}{t} \qquad (3)$$

Their Units are: Displacement (*m*), Speed (ms^{-1}), Velocity (ms^{-1}) and Acceleration (ms^{-2}).

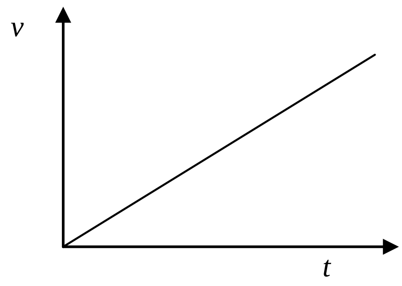
Types of linear motion

(i) Motion with uniform velocity

Velocity-time graphs and displacement-time graphs are valuable ways of depicting motion in a straight line. The figures below shows the displacement- and velocity-time graphs for a body moving with uniform velocity i.e. constant speed in a fixed direction. The slope of fig (a) gives the velocity.



(a) Displacement-time graph



(b) Velocity-time graph

For the displacement-time graph, the slope gives the velocity of the object while the area has no physical significance. In the velocity-time graph, the slope is the acceleration of the object while the area is the displacement of the object.

(ii) Motion with uniform acceleration (equations of motion)

If the velocity of a uniformly accelerating object increases from a value $\vec{u}$ to $\vec{v}$ in time t, then from the definition of acceleration that

$$\vec{a} = \frac{\vec{v} - \vec{u}}{t} \qquad (4)$$

we have

$$\vec{v} = \vec{u} + \vec{a}t \qquad (5)$$

For uniform motion, the displacement covered in time t is defined by

$$\vec{S} = \text{Average velocity} \times \text{Time} = \frac{1}{2}(\vec{u} + \vec{v})t \qquad (6)$$

Using equation 5, equation 6 gives

$$\vec{S} = \vec{u}t + \frac{1}{2}\vec{a}t^2 \qquad (7)$$

Eliminating t in equation (7) by substituting for t in (6) gives

$$v^2 = u^2 + 2aS \qquad (8)$$

Equations 5, 7 and 8 are equations of motion for an object moving in a straight line with uniform acceleration.

Example A body covers a distance of 10*m* in 4*s*, it rests for 10s and finally covers an additional distance of 90*m* in 6*s*. Calculate its average speed.

Solution Total distance $= 10 + 90 + 0 = 100m$

Total time $= 4 + 10 + 6 = 20s$

Average speed $= \dfrac{100m}{20s} = 5ms^{-1}$

Example A student runs 800*m* due north in 110*s* followed by 400*m* due south in 90*s*. Calculate his average speed and his average velocity for the whole journey.

Solution (i) Average speed $= \dfrac{Total\ distance}{Total\ time} = \dfrac{800+400}{110+90} = \dfrac{1200m}{200s} = 6ms^{-1}$

(ii) Average velocity $= \dfrac{Total\ Displacement}{Total\ time} = \dfrac{800-400}{110+90} = 2ms^{-1}$

For velocity, since it is a vector, you have to choose the direction

Example A car moving with a velocity of 54kmhr^{-1} accelerates uniformly at the rate of 2ms^{-2}. Calculate the distance travelled from the place where the acceleration began to that where the velocity reaches 72$kmhr^{-1}$ and the time taken to cover this distance.

Solution Given $54kmhr^{-1} = 15ms^{-1}(\vec{u})$ $\quad 72kmhr^{-1} = 20ms^{-1}(\vec{v})$ $\quad \vec{a} = 2ms^{-2}$, then

(i) $v^2 = u^2 + 2aS$ implying $20^2 = 15^2 + 2(2)S$ $\quad S = 43.75m$

(ii) $\vec{v} = \vec{u} + \vec{a}t$ implying $20 = 15 + 2t$ $\quad t = 2.5s$

(iii) **Motion under gravity (free falling bodies)**

A body released near the earth's surface will accelerate towards the earth under the influence of gravity. If air resistance is neglected, then the body will be in free fall and the motion will proceed with uniform acceleration of $\vec{a} = \vec{g} \approx 9.81ms^{-2}$. The value of this downward acceleration $(\vec{g})$ is the same for all bodies released at the same location and is independent of the bodies' speed, mass, size and shape. The **equations of**

motion for freely falling bodies are similar to those for linear motion with constant acceleration $\underline{\vec{a}}$ being **replaced** by $\underline{\vec{g}}$. For upward motion (rising body), $\vec{g}$ is negative since the body is decelerating. Thus

$$\vec{v} = \vec{u} + \vec{g}t$$
$$\vec{S} = \vec{u}t + \frac{1}{2}\vec{g}t^2 \qquad (9)$$
$$v^2 = u^2 + 2\vec{g}S$$

Example A ball is thrown vertically upwards with a velocity of 20 ms^{-1}. Neglecting air resistance, calculate

(i) The maximum height reached

(ii) The time taken to return to the ground

Solution Taking upward direction as positive, $\vec{u} = 20ms^{-1}$ and $\vec{a} = -\vec{g} = 10ms^{-2}$ then

(i) At maximum height, $v = 0ms^{-1}$ thus

$$v^2 = u^2 + 2aS \Rightarrow 0 = 20\times 20 + 2(-10)S$$

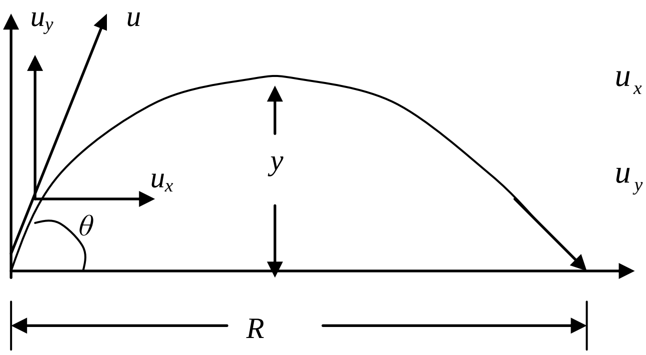
$$S = 20m$$

(ii) On return to ground, *S* becomes zero, thus from

$$S = ut + \frac{1}{2}at^2$$
$$0 = ut - \frac{1}{2}gt^2 = 20t - 5t^2$$
$$t = 4\sec$$

Motion in Two Dimensions (Projectile Motion)

Projectile motion is when an object moves both vertically and horizontally on the surface of the Earth at the same time. The two motions are independent of each other i.e. the object moves horizontally with constant speed, and at the same time, it moves vertically in a way a similar object not undergoing horizontal motion would move. If air resistance is neglected, then a projectile can be considered as a freely falling object and its equations of motion can be determined from the linear equations of motion together with the initial conditions i.e. the initial velocity has components $u\cos\theta$ along the horizontal- and $u\sin\theta$ along the vertical direction. The horizontal and vertical motions are analyzed as follows



$$u_x = u\cos\theta$$

$$u_y = u\sin\theta$$

(i) Vertical motion: The vertical component of *u* is $u\sin\theta$ and the acceleration is –*g*. When the projectile reaches the ground at *B*, the vertical distance *h* travelled is zero. So from

$$h = u_y t - \frac{1}{2}gt^2 \text{ …..} \qquad (1)$$

we have

$$0 = u\sin\theta t - \frac{1}{2}gt^2 \Rightarrow t = 2\frac{u\sin\theta}{g} \text{ …..} \qquad (2)$$

which is the total time of flight. Also the maximum height reached can be evaluated as follows; from equation (1) we have

$$Y = u\sin\theta t - \frac{1}{2}gt^2 = \frac{u^2\sin^2\theta}{2g} \text{…..} \qquad (3)$$

after substituting half of the value of *t* from 2.

(ii) Horizontal motion: Since *g* acts vertically, it has no component in the horizontal direction. So the projectile moves in a horizontal direction with a constant velocity $u\cos\theta$ which is the horizontal component of u. Hence, from the relation $\vec{s} = \vec{u}t$ we have

$$R = u\cos\theta \times \frac{2u\sin\theta}{g} = 2u^2\frac{\sin\theta\cos\theta}{g} = u^2\frac{\sin 2\theta}{g} \dots \qquad ..(4).$$

The maximum range is obtained when $\sin 2\theta = 1$ or $2\theta = 90^o$.

In this case

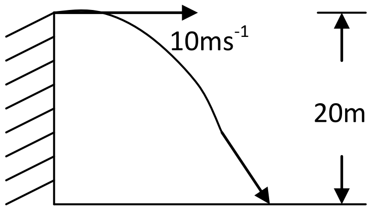
$$R = \frac{u^2}{g} \dots \qquad ..(5).$$

Also at the maximum height *Y* of the path, the vertical velocity of the projectile is zero. So applying the relation $v_y = u_y + at$ in a vertical direction, the time *t* to reach *A* is given by

$$0 = u\sin\theta - gt \Rightarrow t = \frac{u\sin\theta}{g} \qquad .(6).$$

This is just half the time to reach *B*.

Example An object is thrown horizontally with a velocity of 10*m/s* from the top of a 20*m*-high building as shown. Where does the object strike the ground?



Solution: We consider the horizontal and vertical problems separately.

(i) In the vertical case if the downward direction is taken to be positive, then $v_{oy} = 0ms^{-1}$, $\vec{g} = 9.81ms^{-2}$ and $y = 20m$. Thus we can find the time taken to reach the ground from

$$y = v_{oy} + \frac{1}{2}a_y t^2 \Rightarrow 20 = 0 + \frac{1}{2}\times 9.81t^2 \text{ from which}$$

t = 2.02s.

(ii) In the horizontal case, we have found that the object will be in the air for 2.02s. Therefore given that $v_{ox} = v_x = 10ms^{-1}$ and $t = 2.02s$, then $X = v_x t = 10ms^{-1}\times 2.02s = 20.2m$.

Newton's Laws of Motion

First Law: A body tends to remain at rest or in uniform motion in a straight line (with constant velocity) unless acted upon by a resultant force. The tendency of a body to continue in its initial state of motion (a state of rest or a state of uniform velocity) is called inertia. Accordingly, the first law is often called the law of inertia.

Second Law: If a net force acts on a body, it will cause an acceleration of that body. That acceleration is in the direction of the net force and it magnitude is proportional to the magnitude of the net force and inversely proportional to the mass of the body i.e. $\vec{a} \propto \frac{\vec{F}}{m}$ so that $\vec{F} \propto m\vec{a}$. From the definition of a Newton, the law can be written in the form

$$\vec{F} = km\vec{a} = m\vec{a} \qquad (1)$$

This (vector) equation is a relation between vector quantities $\vec{F}$ and $\vec{a}$, and is equivalent to the three algebraic equations

$$\begin{aligned} \vec{F}_x &= m\vec{a}_x \\ \vec{F}_y &= m\vec{a}_y \\ \vec{F}_z &= m\vec{a}_z \end{aligned} \qquad (2)$$

Third Law: Action and reaction are always equal and opposite i.e. when one body exerts a force on another, the second exerts an equal, oppositely directed force on the first. Examples include when pushing on a car, the car pushes back against your hand, when a weight is supported by a rope, the rope pulls down on the hand; a book resting on a table pushes down on the table, and the table in turn pushes up against the book; the earth pulls on the moon holding it in a nearly circular orbit and the moon pulls on the earth causing tides. The law differs from the first and second in that, whereas the first and second laws are concerned with the behaviour of a single body, the third law involves two separate bodies. The inherent symmetry of the action-reaction couple precludes identifying one as action and the other as reaction.

Collisions and Linear Momentum

Linear Momentum is defined as the product of the object's mass (m) and its velocity $\vec{v}$ and is a vector.

$$\text{Linear momentum } \vec{L} = mass \times velocity = m\vec{v}$$

The SI unit of linear momentum is $kgms^{-1}$ (Newton second-Ns) and its dimension is $\frac{[M][L]}{[T]}$. From Newton's second law $(\vec{F} = m\vec{a})$, if no external force acts on an object, then

$\vec{F} = m\vec{a} = m\left(\frac{v-u}{t}\right) = \frac{\Delta\vec{L}}{t} = 0 \Rightarrow \vec{L}$ is a constant. Thus its momentum is conserved. This is the principle of conservation of linear momentum. It is useful in solving problems involving collisions between bodies.

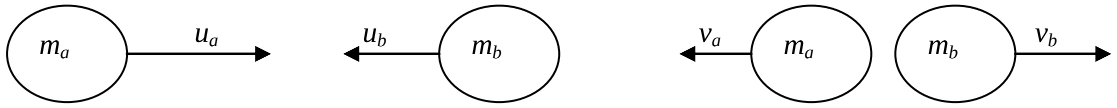
The product of the force and the time is called the impulse of the force I i.e.

Impulse $\vec{I} = \vec{F}t = m\vec{v} - m\vec{u} =$ change in momentum

The SI unit of impulse is the same as that of momentum i.e. Newton-second or kilogram-meter-per-second.

Collision: is any strong interaction between bodies that lasts a relatively short time. Examples include automobile accidents, neutrons hitting atomic nuclei in a nuclear reactor, balls colliding, the impact of a meteor on the surface of earth, a close encounter of a spacecraft with the planet Saturn etc. In all collisions, momentum is conserved. The total energy is also conserved. However, kinetic energy might not be conserved since it might be converted into other forms of energy like sound, heat or work during plastic deformation. There are two main types of collisions: elastic and inelastic collisions.

Elastic collision: both kinetic energy and momentum are conserved.



(a) Before collision (b) After collision

From the figure, we have

$$m_a\vec{u}_a + m_b\vec{u}_b = m_a\vec{v}_a + m_b\vec{v}_b \text{ (conservation of linear momentum)}$$

$$\frac{1}{2}m_a u_a^2 + \frac{1}{2}m_b u_b^2 = \frac{1}{2}m_a v_a^2 + \frac{1}{2}m_b v_b^2 \text{ conservation of kinetic energy.}$$

Inelastic collision: momentum is conserved but kinetic energy is not conserved. Thus

$$m_a\vec{u}_a + m_b\vec{u}_b = m_a\vec{v}_a + m_b\vec{v}_b$$

If the colliding bodies stick together, the collision is totally inelastic and hence we have

$$m_a\vec{u}_a + m_b\vec{u}_b = (m_a + m_b)\vec{V}$$

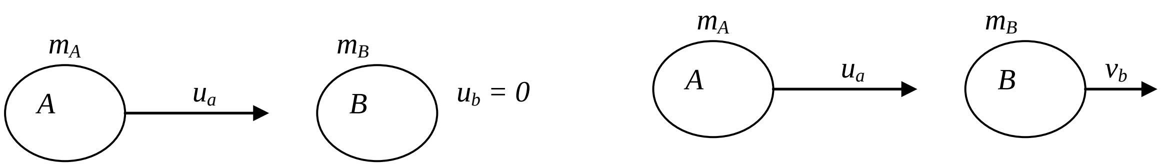
where $\vec{V}$ is the common velocity.

Special cases

(i). Elastic collision in a straight line

If the two objects *A* and *B* have equal masses *m* and mass *B* is stationary ($u_B = 0$) then for elastic collision, we have

$$mu_A = mv_A + mv_B \quad \text{(conservation of linear momentum)}$$



(a) Before collision (b) After collision

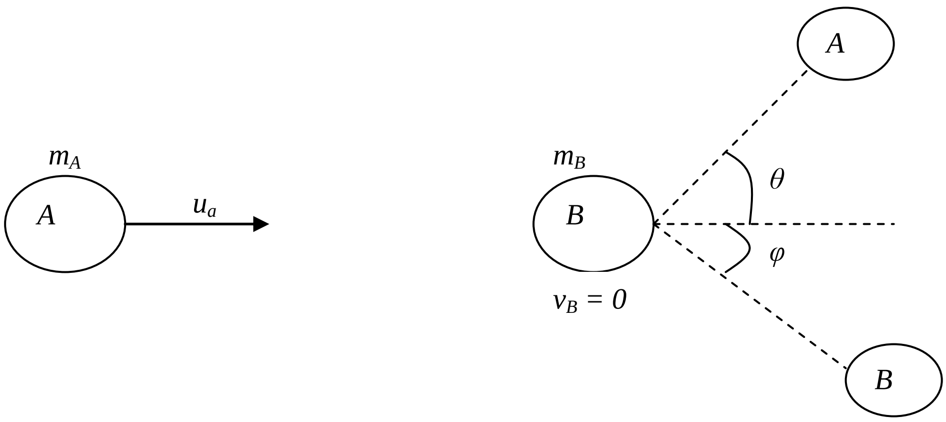
and $\frac{1}{2}mu_A^2 = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2$ conservation of kinetic energy, from which we have

$$u_A = v_A + v_B$$
$$u_A^2 = v_A^2 + v_B^2$$

Solving gives $u_A = v_B$ and $v_A = 0$. Thus the two objects simply exchange velocities i.e. mass *A* comes to rest while mass *B* moves off with the original velocity of *A*. this is a situation of maximum energy transfer between two colliding bodies and is mostly applicable in nuclear reactions where neutrons are stopped by protons.

(ii). Oblique collisions of equal masses

If mass *A* collides obliquely with mass *B* which is at rest and both objects are of equal masses m, then the total momentum of any object will be the sum of the respective momentum components in the vertical and horizontal directions respectively



Thus conservation of linear momentum gives

$$mu_A = mv_A \cos\phi + mv_B \cos\theta \quad \text{(along the } x\text{-direction)}$$

$$0 = -mv_A \sin\phi + mv_B \sin\theta \quad \text{(along the} + y \text{ direction)}$$

Conservation of kinetic energy gives $\frac{1}{2}mu_A^2 = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2$

(iii). **Recoil**

In a case where part of a composite body suddenly flies apart e.g. a bullet fired from a gun, the remaining part (the gun) must undergo momentum in the opposite direction (recoil) in order to conserve the momentum. If m_b and v_b are the mass and velocity of the bullet while m_g is the mass of the gun, then the gun will recoil with velocity v_g given by

$$-m_b v_b = m_g v_g$$

or $v_g = \frac{m_b v_b}{m_g} \Rightarrow v_g$ is far much less than v_b since m_b is far much less than m_g.

Example A car travelling at 90$kmhr^{-1}$ slams into a tree and is stopped in 40*ms*. If the car has a mass of 800*kg*, calculate the average force acting on the car during the collision.

Solution $\vec{v} = 90kmhr^{-1} = 25ms^{-1}$ 1

From $\vec{F}t = m\vec{v} - m\vec{u}$, we have

$$0.04\sec \times F = 800kg \times 25ms^{-1} \Rightarrow F = 5 \times 10^5 N$$

Example A person of mass 50*kg* who is jumping from a height of 5*m* will land on the ground with a velocity

$$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 5} = 10ms^{-1} \text{ for } g \approx 10ms^{-2}.$$

If he does not flex his knees on landing, he will be brought to rest very quickly, say $\frac{1}{10}th$ second. The force *F* acting is then given by $F = \frac{\Delta momentum}{t} = \frac{50 \times 10}{0.1} 5000N$. This is a force of about 10 times the person's weight and the large force has a severe effect on the body. Suppose, however, that the person flexes his knees and is brought to rest much more slowly on landing, say **1** second. Then the force *F* now acting is 10 times less than before, or 500*N*. Consequently, much less damage is done to the person on landing.

Work, Energy and Power

WORK: is defined as the product of the force, the displacement of the point at which the force is applied and the cosine of the angle between the force vector *F* and the displacement vector S. Mathematically:

$$\Delta W = \vec{F}.\Delta\vec{S} = F\Delta S \cos\theta$$

where ΔW is the amount of work done by the force of magnitude *F* during a small displacement of magnitude ΔS . The unit of work is the Newton-meter or kgm^2s^{-2} and is given the name Joule. One joule is the amount of work done by a force of one Newton acting over a distance of one meter in the direction of the displacement.

Features of the definition

First work requires the action of a force. Without a force, no work is done. Second, the application of a force is a necessary but not sufficient condition for work. Work is done only if there is displacement of the point of application of the force, and then only if this displacement has a component along the line of action of the force.

Example Calculate the work done by a man of mass 65*kg* in climbing a ladder 4*m* high.

Solution: Work done = force$\times$distance = weight$\times$ distance = mgh = 65$kg\times$10$ms^{-2}\times$4m=2600J or 2.6kJ

Example How much work is done in lifting a 3*kg* mass a height of 2*m* and in lowering it to its initial position?

Solution: (i) Since the force is directed up and the displacement is in the same direction, then $\theta = 0$. Hence

$$W = mgs = 3kg \times 9.8ms^{-2} \times 2m = 58.8J$$

(ii) Suppose we now slowly lower this mass to its original position. Again we must apply an upward force of *mg* to prevent it from dropping. How much work is done by this force? Now the angle between that force and the displacement is $\theta = 180^o$ and since cos180^o = -1, we have

$$W = -58.8J$$

The negative sign tells us that gravity has done work on the body. In this example, there are two forces that act on the 3*kg* mass: the force of gravity, which points downwards and the tension in the string which pulls upward. If we had asked for the work done by the force of gravity, it would have been negative during the lifting of weight and positive as the weight was lowered.

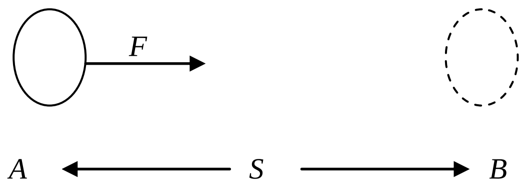
ENERGY: Energy is defined as the capacity to do work. A system may have mechanical energy by virtue of its position, its internal structure or its motion. There are also other forms of energy besides mechanical, namely chemical energy(found in foods, oils, charcoal, biogas etc and is due to the kinetic energy and potential energy of the electrons within atoms), electrical energy(associated with the electric charge and can be produced by generators from hydroelectric power stations-waterfalls, geothermal stations, nuclear fission etc-), nuclear energy from a nuclear reactor, thermal energy (due to heat produced from burning fuels, the sun, heaters etc).

It is a remarkable fact about our physical universe that whenever one form of energy is lost by a body/system, this energy never disappears but it is merely translated into other forms of energy e.g. Vehicles burn fuels to produce both thermal (heat) and mechanical energy.

Mechanical Energy

It is the energy of motion-whether that energy is in action or stored. It exists in two forms:

Kinetic energy- energy possessed by a body by virtue of its motion and it represents the capacity of the body to do work by virtue of its speed. If a force $\vec{F}$ acts on an object of mass m such that the mass accelerates uniformly from initial velocity v_i to a final velocity $v_f\ ms^{-1}$ over a distance S (as shown),



Then the work done over the distance S is

$$W = \vec{F}.\vec{S}$$

But

$$\vec{F} = m\vec{a}$$

and

$$\vec{S} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

From the relation

$$\vec{v} = \vec{u} + \vec{a}t$$

then

$$t = \frac{\vec{v} - \vec{u}}{\vec{a}}$$

so that the work done is

$$W = m\vec{a} \bullet \vec{S} = \frac{1}{2}m\left(v^2 - u^2\right) = \Delta k.e$$

which is the WORK-ENERGY RELATION (THEOREM). If the body starts from rest, then the work done on the object equals kinetic energy gained by the object.

Potential energy-energy possessed by a body by virtue of its configuration (position) in a force field e.g. gravitational field, electrostatic field, magnetic field etc. If an object of mass m is lifted to a height h from the ground, then:

Work done on the mass $W = \vec{F} \cdot \vec{h} = mgh$

i.e. work done on the object $=$ gain in the potential energy by the object.

Whether a body falls vertically or slides down an inclined plane, the work done on it by gravity depends only on its mass and on the difference in height between the initial and final positions. Potential energy of an object depends only on its location and not on the route by which it arrived at that point. It follows that if a body is transported around a closed path, the change in potential energy vanishes i.e. potential energy is independent of the previous history because the gravitational force is conservative. A force is said to be conservative if the work W_{AB} done by the force in moving a body from A to B depends only on the position vectors r_A and r_B. In particular, a conservative force must not depend on time, or on the velocity or acceleration of the body.

Example A 100*kg* crate of milk is pushed up a frictionless 30^o inclined plane to a 1.5*m*-high platform. How much work is done in the process?

Solution The *x* component of *mg* is –mgsin30^o. This force must be balanced by the applied force *F* to prevent the crate from slipping down the plane. The work done by the force *F* is

$$W = Fd\cos\theta$$

Since F acts in the direction of motion, $\theta = 0^o$ and $cos\theta = 1$. The distance d over which the force acts is the length of the incline namely

$$d = \frac{1.5m}{\sin 30} \text{ Hence } W = (mg\sin 30^o)\left(\frac{1.5m}{\sin 30}\right) = 1470J$$

Example Suppose in the previous example the inclined plane is not frictionless and that the coefficient of friction is 0.2. How much work is done in pushing the crate to the 1.5m-high platform?

Solution The work done against the force of gravity is the same as before 1470J. However, the applied force must be greater than $mgsin30$ so as to overcome the force of friction which also acts in the $-x$ direction (opposite to the direction of motion). The force of friction is

$$f_k = \mu_k R = -\mu_k mg\cos 30^o - 0.2(980N)(0.866) = -170N$$

The work done against this frictional force is then

$$W_n = -f_k d = 0.2(980N)(0.866)\left(\frac{1.5m}{\sin 30}\right) = 510J$$

where n indicates nonconservative forces. The total work done in bringing the crate to the platform is

$$W = \Delta pe + W_n = 1470J + 510J = 1980J$$

POWER is the rate of doing work i.e. it is the rate at which energy is converted from one form to another. Mathematically,

$$\overline{P} = \frac{\Delta W}{\Delta t}(average\ power)$$

Also since $\Delta W = \vec{F}.\Delta\vec{S}$, then $\vec{P} = \frac{\Delta W}{\Delta t} = F\frac{\Delta S}{\Delta t} = \vec{F}.\vec{v} = Force \times velocity$

The unit of power is the Watt (W) which is the rate of work (transfer of energy) of one joule per second. Power is also measured in horsepower (hp) where

1hp = 746W

Example A manually operated winch is used to lift a 200kg mass to the roof of a 10m tall building. Assuming that you can work at a steady rate of 200W, how long will it take you to lift the object to the roof? Neglect frictional forces.

Solution The work done equals the increase in potential energy of the 200kg mass, namely

$$W = mgh = 200kg \times 9.8ms^{-2} \times 10m = 19600J$$

Since this work is done at a constant rate of 200W, then

$$200W = \frac{19600J}{\Delta t} \Rightarrow \Delta t = \frac{19600J}{200W} = 98\sec$$

Let us see how large an error may have been made by neglecting the kinetic energy of the mass during the ascent. The average speed of the mass is

$$v = \frac{10m}{98\sec} = 0.102ms^{-1}$$

Kinetic energy during ascent is therefore

$$k.e = \frac{1}{2}mv^2 = \frac{1}{2}(200kg)(0.102ms^{-1}) = 1.04J$$

an amount negligibly small compared with the change of 19600J in potential energy. We can therefore safely neglect this small amount of kinetic energy in the problem's solution.

Properties of Matter

PRESSURE: Pressure is defined as the average force per unit area at the particular region of fluid (liquid or gas) i.e.

$$P = \frac{F}{A}$$

where F is the normal force due to the liquid on the side of area A. At a given point in a liquid, the pressure can act in any direction i.e. pressure is a scalar quantity. If the pressure were not the same, there would be an unbalanced force on the fluid at that point and the fluid would move. Also pressure increases with depth h below the liquid surface and with its density ρ so that

$$P = hg\rho$$

When g is in ms^{-2}, h is in m and ρ is in kgm^{-3}, then the pressure is in Newton per meter squared (Nm^{-2}). The bar is a unit of pressure used in meteorology and by definition,

$$1\ bar = 10^5\ Nm^{-2}$$

The Pascal (Pa) is the name given to a pressure of 1 Nm^{-2}. Thus

$$1 \text{ bar} = 10^5 Pa$$

Pressure is often expressed in terms of that due to a height of mercury (Hg). One unit is the torr (after Torricelli);

$$1\text{torr} = 1mmHg = 133.3Nm^{-2}$$

From $P = hg\rho$ it follows that the pressure in a liquid is the same at all points on the same horizontal level in it. Thus a liquid filling the vessel shown below rises to the same height in each section if *ABCD* is horizontal.

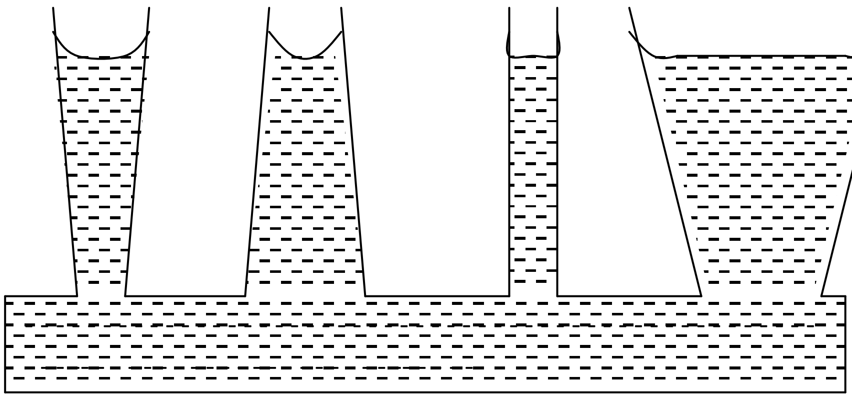
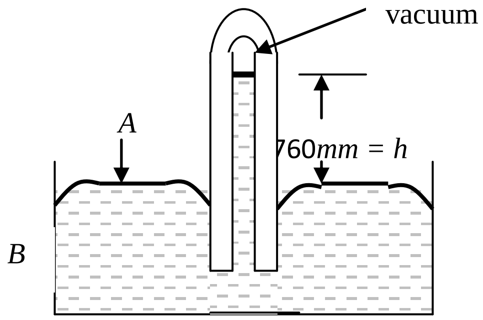
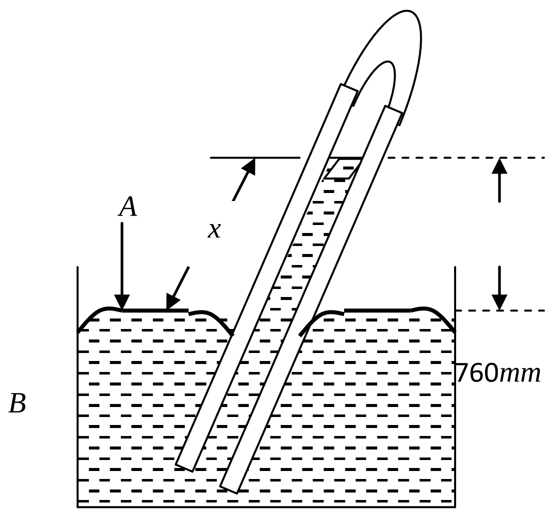


Fig Pressure in a vessel is independent of the cross-section

Atmospheric Pressure

A barometer is an instrument for measuring the pressure of the atmosphere which is required in weather forecasting. It consists of a vertical tube about a meter long containing mercury with a vacuum at the closed top. The other end of the tube is below the surface of mercury contained in a vessel *B*.





The pressure on the surface of the mercury in *B* is atmospheric pressure *A* and since the pressure is transmitted through the liquid, the atmospheric pressure supports the column of mercury in the tube. Suppose the column is a vertical height H above the level of the mercury in *B*. then if $H = 760mm = 0.76m$ and $\rho = 13600kgm^{-3}$, we have

$$P = H\rho g = 0.76 \times 13600 \times 9.8 = 1.013 \times 10^5 Nm^{-2}$$

The pressure at the bottom of a column of mercury 760*mm* high for a particular density and value of *g* is known as standard pressure or one atmosphere. By definition,

$$1 \text{ atmosphere} = 1.01325 \times 10^5 \ Nm^{-2}$$

Standard temperature and pressure is 0^oC and 760*mmHg*. It should be noted that the pressure *P* at a place *X* below the surface of a liquid is given by $P = Hg\rho$ where *H* is the vertical distance of *X* below the surface. In fig ii above, a very long barometer tube is inclined at an angle of 60^o to the vertical. The length of mercury along the slanted side of the tube is x mm say. If the atmospheric pressure is the same as in (i), then the vertical height to the mercury surface is still 760*mm*.

So

$$x\cos 60^o = 760 \Rightarrow x = \frac{760}{\cos 60} = \frac{760}{0.5} = 1520mm$$

ARCHIMEDES' PRINCIPLE

An object immersed in a fluid experiences a buoyant (upthrust) force equal to the weight of the fluid that it displaces.

Application of the principle

If two substances have densities D_1 and D_2, then the density of the second substance relative to the first is

$$\text{Relative density} = \frac{D_2}{D_1}$$

Since $$density = \frac{mass}{volume}$$

then $$D_2 = \frac{m_2}{V_2} \text{ and } D_1 = \frac{m_1}{V_1}$$

If one compares equal volumes of the two substances so that $V_2 = V_1$, then the

$$\text{Relative density } = \frac{D_2}{D_1} = \frac{m_2}{m_1}$$

showing that the relative density will equal the ratio of their masses or their weights. The **density** of a **substance relative** to that of **water** is called the **specific gravity** of the substance i.e. the specific gravity of a substance is equal to the density of the substance divided by the density of water or is equal to the **weight** of a **certain volume** of the **substance divided** by the **weight of** an **equal volume** of **water**. In accurate work it is necessary to specify the temperature at which the measurements are made.

Example A piece of copper suspended from a balance weighs 156.8g in air. When it is completely surrounded by pure water at 20^oC, the reading on the balance is 139.2g. Calculate the specific gravity of copper.

Solution The weight of copper in air = 156.8g

The apparent weight of copper in water = 139.2g

Therefore the buoyant force of the water = 156.8-139.2 = 17.6g

By Archimedes' principle, the weight of the displaced water = 17.6g

The specific gravity of the copper = $$\frac{weight\ of\ Copper}{weight\ of\ equal\ volume\ of\ water}$$

But the volume of the displaced water is equal to the volume of the copper which displaces it. Therefore

The specific gravity of copper = $\frac{156.8g}{17.6g} = 8.91$.

Therefore the density of the copper in this sample is 8.91 times the density of pure water at 20^{o}C. When the specific gravity of a substance is known, its density in any units can be calculated from the known density of water.

Wherever the force of gravity acts on a fluid, the fluid exerts a buoyant force as a result of difference in pressure at different levels. Every fish and submarine in the sea is buoyed up by a force equal to the weight of water displaced. To remain submerged these objects must have a weight equal to or greater than the buoyant force. If they wish to move from one level to another, the balance between the force of gravity and the buoyant force must be disturbed. Some fish can rise by expanding their bodies, thereby displacing more water. Submarines are made to rise by decreasing their weight by forcing water out of their ballast tanks. The air of the earth's atmosphere also exerts a buoyant force on all objects immersed in it e.g. Balloons utilize the buoyancy of air. If a gas such as hydrogen or helium is used to inflate a light-weight plastic envelope, the buoyancy force of the air can be considerably greater than the weight of the balloon. Such balloons are used in making high altitude measurements of various properties of the atmosphere.

Example A weather balloon has a volume of $0.5m^3$ when inflated. The weight of the envelope is 350g. If the balloon is filled with helium, what weight of instrument can it carry a loft? The density of air is $1.29kgm^{-3}$ and the density of helium is 0.138 times the density of air.

Solution By Archimedes' principle, the buoyant force of the air is = the weight of air displaced = the weight of $0.5m^3$ of air

$$=0.5m^3 \times 1.29kgm^{-3} = 0.65kg = 650g$$

Since the relative density of helium is 0.138(air $\approx$ 1), the weight of helium in the balloon is

$$= 0.138 \times 650g = 90g$$

Therefore the weight of the balloon and helium

$$= 350g + 90g = 440g$$

The excess of the buoyant force over the force of gravity

$$= 650g - 440g = 210g$$

Therefore if an unbalanced force of 10g is left available to produce upward acceleration, the weight of the instrument load can be 200g.

Floating at the surface of liquids

According to Archimedes' principle, if a liquid is displaced by a solid, the liquid exerts a buoyant force on the solid. This is true for any fraction of the solid that displaces liquid e.g. when a boy is lifting a stone out of water, he does not have to support the full weight of the stone until it is completely clear of the water. When half of the stone's volume is submerged, the buoyant force is half of the buoyant force when it is completely submerged. Suppose that an object is placed in a liquid of density greater than that of the object. The buoyant force of the liquid will equal the force of gravity before the solid is completely submerged. To go beyond this point of equilibrium, requires then an extra downward force to be applied. If it is not, the object remains in equilibrium under the action of balanced forces: it floats. When an object that will float is placed into a liquid, it will sink into the liquid until the weight that it displaces is equal to its own weight. This relation, a direct consequence of Archimedes' principle, is sometimes called the law of flotation.

Example A block of wood weighing 120*g* has a volume of $180cm^3$. What fraction of its volume would be submerged when floating in alcohol of density $0.80gcm^{-3}$?

Solution For the block to float, it must displace 120*g* of alcohol. The volume of 120*g* of alcohol

$$= \frac{120g}{0.80gcm^{-3}} = 150cm^3$$

To displace $150cm^3$ of alcohol, the block will have to have $150cm^3$ of its volume submerged.

The fraction submerged $= \frac{150}{180} = \frac{5}{6}$

Archimedes' principle provides the basis for the design of ships made of steel. For a steel vessel to float it is only necessary to spread the steel around so that it can displace an amount of water having a weight that exceeds the weight of the steel.

Example A steel box is constructed to make a cube 10*cm* on a side, from material 0.20*cm* thick. What weight of contents is possible before the box sinks in a liquid of specific gravity 1.2? The density of steel is $7.0gcm^{-3}$

Solution The volume of the box is $(10cm)^3 = 1000cm^3$

The maximum buoyant force that the liquid can provide is $1000cm^3 \times 1.2 \times 1.0gcm^{-3} = 1200g$

The weight of the box which has 6 sides, each 10*cm* square and 0.2*cm* thick, and which is made of steel of density $7gcm^{-3}$ is

$$= 6 \times 10cm \times 10cm \times 0.2cm \times 7gcm^{-3} = 840g$$

The excess of buoyant force over weight is

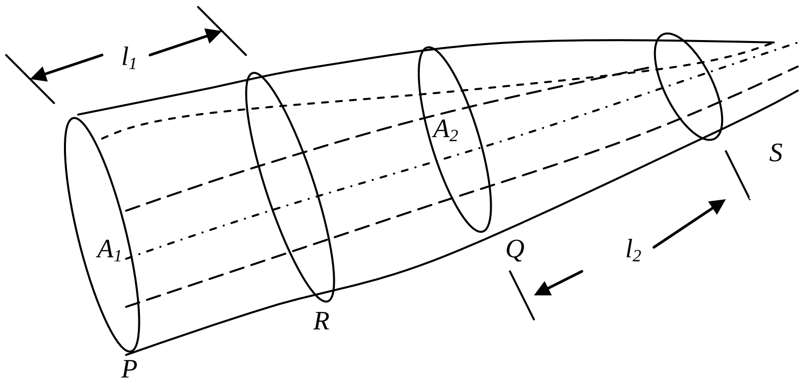
$$1200g - 840g = 360g$$

If material is added to the box, it will continue to float until the weight of the contents exceeds 3.6×10^2g.

Hydrometers

The relation between fluid density and floating provides basis for the construction and use of hydrometers. Hydrometers usually consist of a hollow tube weighted at one end and having a graduated scale at the other end. In a liquid, the weighted end ensures that the instrument floats upright. The depth to which it sinks will depend on the density of the fluid. The higher the density of the liquid, the greater will be its buoyant force per unit volume of the hydrometer immersed. When the scale is graduated using liquids of known density, the hydrometer may be used to measure the density of unknown liquids. A number of specialized hydrometers are used for the determination of specific gravity: in dairies for milk, in automobile service stations for antifreeze and battery acids, in chemical laboratories for determining the composition of aqueous solutions (in a 12.5% sugar solution at 13^oC, a hydrometer would indicate a specific gravity of 1.05).

Fluids in motion: The figure below shows a tube of water flowing steadily between *X* and *Y* where *X* has a bigger cross-sectional area A_1 than the part of cross-sectional area A_2. The streamlines of the flow represent the directions of the velocities of the particles of the fluid and the flow is uniform or laminar.



Assuming the liquid is incompressible, then if it moves from *PQ* to *RS*, the volume of liquid between *P* and *R* is equal to the volume between *Q* and *S*. Thus

$$A_1 l_1 = A_2 l_2 \Rightarrow \frac{l_2}{l_1} = \frac{A_1}{A_2}$$

where l_1 is *PR* and l_2 is *QS*. Hence l_2 is greater than l_1. Consequently the velocity of the liquid at the narrow part of the tube, where the streamlines are closer together, is greater than at the wider part *Y* where the streamlines are further apart. For the same reason, slow-running water from a tap can be made into a fast jet by placing a finger over the tap to narrow the exit.

Bernoulli's principle

Bernoulli obtained a relation between the pressure and velocity at different parts of a moving incompressible fluid. If viscosity is negligibly small, there are no frictional forces to overcome. Hence the work done by the pressure difference per unit volume of a fluid flowing a long a pipe steadily is equal to the gain in kinetic energy per unit volume plus the gain in potential energy per unit volume.

The work done by a pressure in moving a fluid through a distance *W* = force $\times$ distance moved = pressure $\times$area $\times$distance moved = pressure$\times$ volume.

At the beginning of the pipe where the pressure is P_1, the work done per unit volume on the fluid is P_1; at the other end the work done per unit volume is P_2. Hence the net work done on the fluid per unit volume

$$\Delta W = P_1 - P_2.$$

The kinetic energy per unit volume = $\frac{1}{2}$ mass per unit $\times$velocity^2 = $\frac{1}{2}\rho\times$velocity^2

where ρ is the density of the fluid. Thus if v_2 and v_1 are the final and initial velocities respectively at the end and the beginning of the pipe, the kinetic energy gained per unit volume

$$\Delta k.e = \frac{1}{2}\rho({v_2}^2 - {v_1}^2)$$

Further, if h_2 and h_1 are the respective heights measured from a fixed level at the end and beginning of the pipe, the potential energy gained per unit volume = mass per unit volume $\times$g $\times$ $\left(h_2 - h_1\right)$

or

$$\Delta p.e = \rho g\left(h_2 - h_1\right)$$

Thus from the conservation of energy

$$P_1 - P_2 = \frac{1}{2}\rho(v_2{}^2 - v_1{}^2) + \rho g(h_2 - h_1)$$

$$P_1 + \frac{1}{2}\rho v_1{}^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2{}^2 + \rho g h_2$$

Therefore

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{Constant}$$

where P is the pressure at any part and v is the velocity there. Hence, for streamline motion of an incompressible non-viscous fluid,

> **The sum of the pressure at any part plus the kinetic energy per unit volume plus the potential energy per unit volume there is always a constant.**

This is known as Bernoulli's principle. The principle shows that at points in a moving fluid where the potential energy change $\rho g h$ is very small, or zero as in flows through a horizontal pipe, the pressure is low where the velocity is high; conversely, the pressure is high where the velocity is low.

Example As a numerical illustration, suppose the area of cross-section A_1 of X in the figure above is $4cm^2$, the area A_2 of Y is $1cm^2$ and water flows past each section in laminar flow at the rate of $400cm^3s^{-1}$, then

at X speed v_1 of water = $\dfrac{volume\ per\ \sec ond}{area} = \dfrac{400cm^3s^{-1}}{4cm^2} = 100cms^{-1} = 1ms^{-1}$

at Y speed v_2 of water= $\dfrac{400cm^3s^{-1}}{1cm^2} 400cms^{-1} = 4ms^{-1}$

The density of water $\rho = 1000kgm^{-3}$. So if P is the pressure difference, then

$$P = \frac{1}{2}\rho\left(v_2{}^2 - v_1{}^2\right) = \frac{1}{2}\times 1000 \times \left(4^2 - 1^2\right) = 7.5\times 10^3\ Nm^{-2}$$

Therefore $\qquad P = hg\rho \Rightarrow h = \dfrac{P}{\rho g} = \dfrac{7.5\times 10^3}{1000\times 9.8} \approx 0.77m$

The pressure head is thus equivalent to 0.77m of water.

Applications of Bernoulli's principle: 1. A suction effect is experienced by a person standing close to the platform at a station when a fast train passes. The fast-moving air between the person and the train produces a decrease in pressure and excess air pressure on the other side may push the pedestrian towards the train.

2. Filter pump. A filter pump has a narrow section in the middle, so that a jet of water from the tap flows faster here. This causes a drop in pressure near it and air therefore flows in from the side tube to which a vessel is connected. The air and water together are expelled through the bottom of the filter pump.

3. Aerofoil lift. The curved shape of an aerofoil creates a faster flow of air over its top surface than the lower one. This is shown by the closeness of the streamlines above the aerofoil compared with those below. From Bernoulli's principle, the pressure of the air below is greater than that above, and this produces the lift on the aerofoil.

4. Flow of a liquid from wide tank. Consider the figure below. At the top X of the liquid in the tank, the pressure is atmospheric say B, the height measured from a fixed level such as the hole H is h, and the kinetic energy is negligible if the tank is wide so that the level falls very slowly. At the bottom, Y near H, the pressure is again B, the height is zero and the kinetic energy is $\frac{1}{2}\rho v^2$ where ρ is the density and *v* is the velocity of emergence of the liquid. Thus from Bernoulli's principle,

$$B + \rho gh = B + \frac{1}{2}\rho v^2 \Rightarrow v = \sqrt{2gh}$$

Thus the velocity of the emerging liquid is the same as that which would be obtained if it fell through a height h and this is Torricelli's theorem. In practice the velocity is less than that given by $\sqrt{2gh}$ owing to viscous forces and the lack of streamline flow.

Example Water flows steadily a long a horizontal pipe at a volume rate of $8\times10^{-2}m^3s^{-1}$. If the area of cross-section of the pipe is $40cm^2$, calculate the flow velocity of the water. Find the total pressure in the pipe if the static pressure in the horizontal pipe is $3.0\times10^4\,Pa$, assuming the water is incompressible, non-viscous and its density is $1000kgm^{-3}$. What is the new flow velocity if the total pressure is $3.6\times10^4\,Pa$.

i Velocity of water= $\frac{volume\ per\ second}{area} = \frac{8\times10^{-3}}{40\times10^{-4}} = 2ms^{-1}$

ii Total pressure=static pressure $+\frac{1}{2}\rho v^2 = 3.0\times10^4 + \frac{1000\times2^2}{2} = 3.2\times10^4\,Pa$

iii $\frac{1}{2}\rho v^2 =$ total pressure- static pressure

Therefore $\frac{1}{2}\times1000\times v^2 = 3.6\times10^4 - 3.0\times10^4 = 0.6\times10^4$

$$v = \sqrt{\frac{0.6\times10^4}{500}} = 3.5ms^{-1}.$$

OSCILLATIONS AND WAVES

If a stone is thrown into a pool of water, circular ripples which spread outwards from the point where the stone entered the water are observed. The same case happens when one end of the rope is moved up and down in a direction perpendicular to its length. These ripples indicate the transfer of energy from the stone through the water and are known as waves. In this context, a wave can be defined as an oscillating motion by which energy is transmitted from one point to another. Also, a wave can be defined as a disturbance that transfers energy from one point to another. For the rope, the particles near the end exert a drag on their neighbour which then begins to oscillate as well. Finally, each particle is oscillating up and down slightly later than the one immediately before it. The net result is the appearance of a series of equidistant crests and troughs which travel with velocity called the wave velocity as in the figure below:

Definition of Terms

Trough and Crest: Crests are the high parts of a wave while troughs are the low parts.

Wavelength (λ): is the distance between any two successive crests or troughs that represents a complete oscillation. Wavelengths can also be measured between any two successive particles that are exactly the same position in their paths and moving in the same direction. From the figure, we have the following wavelengths: AE, XY BF, and EI.

Amplitude (*a*): is a measure of maximum or minimum displacement a particle moves from its rest position when disturbed by a wave.

Period (*T*): is the time taken by a particle in a wave disturbance to make one complete oscillation or one wavelength.

Frequency (*f*): refers to the number of complete oscillations produced per unit time. The SI unit of frequency is the Hertz (*Hz*) or (s^{-1})

Relationship between velocity (*v*), wavelength (λ) and frequency (*f*)

Since the period of a wave is the time required for a complete wave to pass a given point and a complete wave is the wavelength, then the speed of the wave can be given by

$$Speed = \frac{Distance}{Time}$$

Or $$v = \frac{\lambda}{T}$$

This can be factorized to yield

$$v = \frac{1}{T}\lambda$$

But $$\frac{1}{T} = f$$

Hence, $$v = \lambda f$$

Example: A rope is displaced at a frequency of 3*Hz*. If the distance between successive crests of the wave train is 0.8*m*, calculate the speed of the waves along the rope.

Solution: Since $f = 3Hz$ and $\lambda = 0.8m$, then

$$v = \lambda f = 3Hz \times 0.8m = 2.4ms^{-1}$$

Example: Consider the figure below.

If the speed of the wave is $2.4ms^{-1}$, calculate the wave's

(a) Amplitude (a)

(b) Frequency (f)

(c) Wavelength (λ)

Solution: Amplitude $a = 0.4m$, Frequency $f = 5Hz$, Wavelength $\lambda = 0.4m$.

Types of wave motion

1. **Transverse waves**: is a wave whereby the particles of the medium vibrate about their mean positions in the direction perpendicular to the direction of propagation as in the figure below. Examples include Electromagnetic waves (light waves, *x*-rays radio wave etc), water waves, waves on strings or ropes, seismic waves etc.

2. **Longitudinal Waves:** particles of the medium vibrate about their mean position in the direction of the wave or parallel to the direction of the wave, e.g. sound wave. They are best demonstrated using a spring. When several turns of a spring are pinched together at one end, they create a compression. On the other hand, if pulled apart, they create a rarefaction as in the figure below:

Both transverse and longitudinal waves are said to be progressive waves i.e. the wave profile moves along with the speed of the wave. If a snapshot is taken of a progressive wave, it repeats at equal distances. The repeat distance is the wavelength.

BASIC OPTICS (LAWS OF GEOMETRICAL OPTICS)

The properties of light that are most important to the design of various optical instruments can be summarized into the laws of reflection, refraction and dispersion.

The Laws of Reflection

The law of reflection describes the manner in which light is reflected from a smooth, shiny surface. If light strikes a shiny surface, its direction must make a certain angle with the normal to the surface at the point where it strikes. That angle is the angle of incidence and the angle the reflected beam makes with the normal is called the angle of reflection. In particular, if a ray of light, AO is incident on a plane mirror XY at O, the angle AON made with the normal ON to the mirror is the angle of incidence *i* and BON is the angle of reflection *r* as in the figure below:

Experiments show that:

1. The reflected ray, the incident ray and the normal to the mirror at the point of incidence all lie in the same plane.

2. The angle of incidence = the angle of reflection.

These are the two laws of reflection.

The normal to a surface at the point O is simply a line or direction perpendicular to that surface at that point.

Deviation of Light by a Plane Mirror

An instrument such as the periscope uses a plane mirror to change or deviate light from one direction to another. To see this, we consider a ray AO incident at O on a plane mirror XY below:

The angle AOX made by AO with XY is known as the glancing angle *g*, and since the angle of reflection is equal to the angle of incidence, then the glancing angle BOY made by the reflected ray OB with the mirror is also equal to *g*.

The light has been deviated from a direction AO to a direction OB, and since angle COY = angle XOA = *g*, it follows that

angle of deviation, $d = 2g$

In general, the angle of deviation of a ray by a plane surface is twice the glancing angle.

Deviation of Reflected Ray by Rotated Mirror

Consider a ray AO incident at O on a plane mirror M_1, *a* being the glancing angle with M_1 as in the figure below. If OB is the reflected ray, then the angle of deviation COB = $2g = 2a$. Suppose the mirror is rotated through an angle θ to a position M_2, the direction of the incident ray AO being constant. The ray is now reflected from M_2, in a direction OP, and the glancing angle with M_2 is (a + θ). Hence the new angle of deviation COP = $2g$ = 2 (a + θ). The reflected ray has thus been rotated through an angle BOP when the mirror rotated through an angle θ

Since $$B\hat{O}P = C\hat{O}P - C\hat{O}B$$

Then $$B\hat{O}P = 2(a+\theta) - 2a = 2\theta$$

Thus, if the direction of an incident ray is constant, the angle of rotation of the reflected ray is twice the angle of rotation of the mirror. If the mirror rotates through 4^o, the direction of the incident ray kept unaltered, the reflected ray turns through 8^o.

The Laws of Refraction

When a ray of light *AO* is incident at *O* on the plane surface of a glass medium, some of the light is reflected from the surface along *OC* in accordance with the laws of reflection, while the rest of the light travels along a new direction, *OB* in the glass as shown:

On account of the change in direction the light is said to be refracted on entering the glass and *r*-angle of refraction-is the angle made by the refracted ray *OB* with the normal at *O*. Snell, a Dutch physicist, discovered in 1620 that the sines of the angles bear a constant ratio to each other. In particular, he demonstrated that

1. The incident ray, the refracted ray and the normal at the point of incidence, all lie in the same plane.

2. For two given media, $\frac{\sin i}{\sin r}$ is a constant, where i is the angle of incidence and r is the angle of refraction. The constant ratio is known as the refractive index of the two given media.

The law imply that, if light passes from one medium into a second one of a different index of refraction, the angle the light beam makes with the normal to the interface between the two substances is always less in the medium of higher index. Thus, if light goes from air into glass or water, it is bent toward the normal to the interface, while if it goes from water or glass into air, it is bent away from the normal.

So, every transparent material can be characterized by its index of refraction n-a measure of the degree to which the speed of light is diminished in passing through it. Specifically, the absolute index of refraction is the ratio of the speed of light in a vacuum to that in the substance i.e.

$$n = \frac{c}{v}$$

where c is velocity of light in vacuum and v is velocity of light in medium. From the definition of refractive indices in terms of velocities as

$$\frac{v_1}{v_2} = \frac{\sin\theta_1}{\sin\theta_2}$$

Snell's law can be rewritten as

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

From this equation, it follows that if $n_2 > n_1$ then $\sin\theta_2 < \sin\theta_1$ and θ_2 will be smaller than θ_1. In other words, as light travels from a medium of a low index of refraction to one of a high index, it is bent toward the normal. Conversely, light is refracted away from the normal as it passes from a medium of large refractive index to a medium of smaller refractive index. Usually, media of higher densities have higher indices of refraction.

Example: A beam of light is incident on a certain type of glass ($n = 1.51$) as shown in the figure. What is the angle that the refracted ray makes with the plane of the glass surface?

Solution: Let air be medium 1 (n_1 = 1), glass be medium 2 (n_2 = 1.51). The angle of incidence θ_1 is 60^o. From the equation,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

we have

$$\theta_2 = \sin^{-1}\left[\left(\frac{n_1}{n_2}\right)\sin\theta_1\right] = \sin^{-1}\left[\left(\frac{1}{1.51}\right)\times 0\cdot 866\right] = 35^o$$

The angle θ_2 is the angle of refraction, that is, the angle between the refracted ray and the normal to the glass surface. The angle between the refracted ray and the surface is

$$\theta = 90^o - \theta_2 = 90^o - 35^o = 55^o$$

Example: A diver student looks up from inside the water and sees the sun at an angle of 30^o with the zenith (vertical). To an observer above water, what is the angle the sun's rays make with the zenith?
Solution: If we designate air medium 2 (n_2 = 1) and water medium 1 (n_1 = 1.33), then from the equation

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

we have

$$1 \times \sin\theta_2 = 1.33 \times \sin 30^o = 0 \cdot 665$$

$$\theta_2 = 41 \cdot 7^0$$

That is, the sun is 41.7^o below the zenith or 48.3^o above the horizon.

ELECTRICITY AND MAGNETISM

Electrostatics

Positive and Negative Charges

It is common practice that when two substances, such as amber and fur or a plastic ball pen and hair, are rubbed together both are left with the ability to attract light objects. The reason for this phenomenon is that the rubbing action causes electrons to transfer from one substance to the other; the substance gaining electrons acquires a negative charge while the substance losing electrons ends up acquiring a positive charge. Moreover, it is found that when two objects having like electrical charges are freely suspended close to one another, they repel while when unlike charges are similarly suspended they attract. These are the basic laws of electrostatic repulsion and attraction respectively.

The Law of Electrostatic Force between Two Charges

The magnitude of the force between two electrically charged bodies was studied by Coulomb in 1875. He showed that, for two charged bodies, the force *F* was inversely proportional to the square of the distance *r*, i.e.

$$F \alpha \frac{1}{r^2} \qquad 1$$

This result is known as the inverse square law. Also, by measuring the force *F* between two charges when their respective magnitudes Q_1 and Q_2 are varied, he found that *F* is directly proportional to the product Q_1Q_2. Thus

$$F \alpha Q_1 Q_2 \qquad 2$$

Combining equations (1) and (2) yields

$$F \alpha \frac{Q_1 Q_2}{r^2}$$

Or

$$F = k\frac{Q_1 Q_2}{r^2} \qquad 3$$

where *k* is a constant given by $$k = \frac{1}{4\pi\varepsilon_o} \qquad 4$$

Consequently, $$F = \frac{1}{4\pi\varepsilon_o}\frac{Q_1 Q_2}{r^2} \qquad 5$$

where ε_o is a constant called the permittivity of free space if the charges are situated in a vacuum. In this expression, *F* is measured in Newton (*N*), *Q* in coulomb (*C*) and *r* in meter (*m*). Hence, from equation (5), we have

$$\varepsilon_o = \frac{Q_1 Q_2}{4\pi F r^2} \qquad 6$$

The unit of ε are coulomb-squared per Newton per metre-squared ($C^2N^{-1}m^{-2}$).

If the charges are situated in other media such as water, then the force between the charges is reduced. In particular, the force becomes

$$F = \frac{1}{4\pi\varepsilon}\frac{Q_1 Q_2}{r^2} \qquad 7$$

where ε is the permittivity of the medium. For example, the permittivity of water is about eighty times that of a vacuum. This means that the force between charges situated in water is eighty times less than if they were situated the same distance apart in a vacuum.

Example: (a) Calculate the value of two equal charges if they repel one another with a force of 0.1*N* when situated 50*cm* apart in a vacuum.

(b) What would be the size of the charges if they were situated in an insulating liquid whose permittivity was ten times that of a vacuum?

Solution: **(a)** Since $F = \frac{1}{4\pi\varepsilon_o}\frac{Q_1 Q_2}{r^2}$ and $Q_1 = Q_2$, then

$$0.1N = \frac{9\times10^9 Q^2}{(0.5)^2}$$

Or $$Q^2 = \frac{0.1\times(0.5)^2}{9\times10^9}$$

Therefore, $$Q = 1.7\times10^{-6}C$$
$$= 1.7\mu C$$

(b) The permittivity of the liquid is $\varepsilon = 10\varepsilon_o$

$$F = \frac{1}{4\pi\varepsilon}\frac{Q_1 Q_2}{r^2} = \frac{Q_1 Q_2}{4\pi(10\varepsilon_o)r^2} = 9\times10^9\times\frac{Q_1 Q_2}{10r^2}$$

Since the charges are equal and the same force is to be maintained, then

$$Q = \sqrt{\frac{0.1 \times (0.5)^2}{9 \times 10^8}} = 5.27 \times 10^{-6} C = 5.27 \mu C$$

Electric Field Intensity or Field Strength: An electric field can be defined as a region where an electric force is experienced. Arrows on the lines of force show the direction of the force on a positive charge; the force on a negative charge is in the opposite direction. The figure below shows the lines of force, also called electric flux in some electrostatic fields.The force exerted on a charged body in an electric field depends on the charge of the body and on the intensity or strength of the field. The intensity E of an electrostatic field at any point is defined as the force per unit charge which it exerts at that point. Its direction is that of the force exerted on a positive charge. From this definition, it follows that;

$$E = \frac{F}{Q'} \qquad 1$$

and

$$F = EQ' \qquad 2$$

Since F is measured in newtons and Q' in coulombs, then intensity E has units of newtons per metre (NC^{-1}) or a more practical unit of E is volt per metre (Vm^{-1}).

If the test charge Q' is situated at the point P as in the figure below, then the electric field strength at that point is given by;

$$E = \frac{F}{Q'} = \frac{Q}{4\pi\varepsilon_o r^2} \qquad 3$$

The direction of the field is radially outward if the charge Q is positive and is radially inward if the charge Q is negative. If the charge were surrounded by a material of permittivity ε then

$$E = \frac{Q}{4\pi\varepsilon r^2} \qquad 4$$

Example: An electron of charge 1.6×10^{-19} C is situated in a uniform electric field of intensity $120kVm^{-1}$. Find

(i) The force on the charge

(ii) Its acceleration

(iii) The time the charge takes to travel 20mm from rest ($m_e = 9.1 \times 10^{-31}$ kg)

Solution: (i) Force on the electron

$$F = eE = 1.6 \times 10^{-19} C \times 120 \times 10^3 Vm^{-1} = 1.92 \times 10^{-14} N$$

(ii) Its acceleration $a = \frac{F}{m_e} = \frac{1.92 \times 10^{-14} N}{9.1 \times 10^{-31} kg} = 2.12 \times 10^{16} ms^{-2}$

(iii) Time to travel 20mm is

$$S = ut + \frac{1}{2}at^2$$

Since it starts from rest, then u = 0. Hence

$$t = \sqrt{\frac{2S}{a}} = \sqrt{\frac{2 \times 0.02m}{2.12 \times 10^{16} ms^{-2}}} = 1.37 \times 10^{-9} s$$

Electric Flux from a point charge

The density of the lines increases near the charge where the field intensity is high. The intensity *E* at a point can thus be represented by the number of lines per unit area through a surface perpendicular to the lines of force at the point considered and **flux** is the name given to the product of *E* and the area i.e.

$$\Phi = E \times Area \qquad 5$$

This is illustrated in figure (i) below. Consider a sphere of radius *r* drawn in space concentric with a point charge as in figure (ii). The value of *E* at this place is given by equation (4) and the total flux through the sphere is

$$\begin{aligned} \Phi &= E \times Area \\ &= \frac{Q}{4\pi\varepsilon r^2} \times 4\pi r^2 \\ &= \frac{Q}{\varepsilon} \qquad 6 \\ &= \frac{charge\ inside\ sphere}{permittivity} \end{aligned}$$

This demonstrates the important fact that the total flux crossing any sphere drawn outside and concentrically around a point charge is constant. It does not depend on the distance from the charged sphere as long as the inverse square law is obeyed. However, suppose some other force law were valid where

$$E = \frac{Q}{4\pi\varepsilon r^n} \qquad 7$$

Then the total flux through the area is

$$\Phi = \frac{Q}{4\pi\varepsilon r^n} \times 4\pi r^2 = \frac{Q}{\varepsilon} r^{(2-n)} \qquad 8$$

This is only independent of *r* if *n = 2*. Thus the total flux passing through any closed surface whatever its shape is always equal to

$$\Phi = \frac{Q}{\varepsilon} \qquad 9$$

where *Q* is the total charge enclosed by the surface. This relation is known as **Gauss' Law** or Theorem.

CAPACITORS: When two conductive plates are isolated from one another and are connected to the opposite poles of an electrical supply, as in the figure below, an electrostatic field is found to exist between them (this is analogous to the magnetic field produced by an electromagnet). The presence of the electrostatic field can be detected if the plates are supported in an insulating material and mica dust sprinkled over the insulator whilst a voltage of several kilovolts is applied between the plates.

The mica dust arranges itself in the type of pattern shown in the figure. In the SI system of units, one unit of electrostatic flux emanates from a unit charge of one coulomb, so that Q lines of flux emanate from a charge of Q coulombs. Hence,

$$\text{Electric Flux } = Q \ \ coulombs\,(C) \tag{1}$$

The two-electrode structure in the figure above is known as a parallel-plate capacitor, and has the property of storing energy in its electric field (the energy is stored in the region between and around the plates-the majority of the energy being stored between the plates, where the field is strongest). The insulating material between the plates (which could be air, paper, mica, plastic etc) is known as a dielectric. The process of storing energy in the capacitor is known as charging the capacitor, while that of extracting energy from the capacitor is known as discharging the capacitor.

The Capacitance of a Capacitor: The capacitance (*C*) of a capacitor is a measure of the ability of a capacitor to store electric charge. The unit of capacitance is the farad (*F)*. Other units of capacitance are

1 microfarad ($1\mu F$) = 10^{-6} F

1 nanofarad ($1nF$) = 10^{-9} F

1 picofarad ($1pF$) = 10^{-12} F

Experimentally, the relationship between the charge stored (in coulomb), the capacitance (in farads) and the voltage (in volts) across a capacitor is given by

$$Q = CV \tag{1}$$

The unit of capacitance can then be defined from the equation as: A capacitor having a capacitance of one farad stores a charge of one coulomb when a potential difference of one volt appears between its terminals.

Energy Stored in a Capacitor

When charging current flows into a capacitor, energy is stored in its electric field. This energy is restored to the electric circuit when the capacitor is discharged. The expression for the average energy stored in the capacitor in the time interval Δt when the voltage across the capacitor uniformly increases from zero to V volts is given as

$$Average\,energy\,W = Average\,Voltage\ across\,the\,capacitor \times Average\,charging\ current \times Time \tag{1}$$

Since the capacitor voltage increases uniformly from zero to V volts, the average voltage across the capacitor during the time interval Δt is $\frac{V}{2}$. The average current flowing in the capacitor is given by

$$Average\ \ current = C \times change\ \ in\ \ voltage/time$$

$$= C \times \frac{V}{\Delta t} \tag{2}$$

Substituting the above into equation (*1*) gives

Average energy $W = \frac{V}{2} \times C \frac{V}{\Delta t} \times \Delta t$

Hence, $$W = \frac{1}{2} CV^2 \quad J \tag{3}$$

Other versions of equation (*2*) can be obtained as follows. Since

$$Q = CV$$

then $$W = \frac{1}{2} QV \ J$$

Also, since $$V = \frac{Q}{C}$$

Then $$W = \frac{Q^2}{2C} \ J \tag{4}$$

Example: Determine the energy stored in a 10 μF capacitor when the voltage between its terminals is *50V*.

Solution:

Example: Calculate the capacitance of a capacitor if it stores a charge of 2 μC when the energy supplied to it is 10 μJ .

Solution:

Capacitance of a Parallel-plate Capacitor

A parallel-plate capacitor is shown below:

Suppose that all the electric flux passes through the dielectric between the two plates, i.e. none of the flux fringes around the outside of the capacitor. If the area of the dielectric (i.e. the plate area) is a m^2, then the electric flux density D in the dielectric is

$$D = \frac{Q}{a} \ (Cm^{-2}) \tag{5}$$

where Q is the charge stored by the capacitor. The electric field intensity E in the dielectric is

$$E = \frac{V}{d} \ (Vm^{-1}) \tag{6}$$

where V is the voltage between the plates of the capacitor and d is the thickness of the dielectric in metres. Since

$$D = \varepsilon E (Cm^{-2}) \quad (7)$$

then

$$\frac{Q}{a} = \varepsilon \frac{V}{d} (Cm^{-2})$$

But

$$Q = CV$$

hence

$$\frac{CV}{a} = \varepsilon \frac{V}{d} (Cm^{-2})$$

Therefore, the capacitance of the parallel-plate capacitor is

$$C = \frac{\varepsilon a}{d} = \frac{\varepsilon_0 \varepsilon_r}{d} a \ (F) \quad (8)$$

The above equation tells us that:

- Increasing the permittivity ε gives a proportional increase in capacitance i.e. a capacitor with, say, mica or paper as a dielectric has a greater capacitance than a similar capacitor with air as the dielectric.
- Increasing the area of the plates gives a proportional increase in capacitance
- Increasing the distance between the plates reduces the capacitance (the capacitance is inversely proportional to the distance between the plates)

ARRANGEMENT OF CAPACITORS

In radio circuits, for example, capacitors often appear in arrangements whose resultant capacitances must be known. To derive expressions for these, we need the equation defining capacitance in its three possible forms:

$$C = \frac{Q}{V}, \quad V = \frac{Q}{C}, \quad Q = CV \quad 1$$

Parallel Connection of Capacitors

The figure below shows three capacitors, having all their left-hand plates connected together, and all their right-hand plates likewise. They are said to be connected in parallel. If a cell is not connected across them, they all have the same potential difference V (for if they had not, current would flow from one to another until they had). The charges on the individual capacitors are respectively.

$$Q_1 = C_1 V$$

$$Q_2 = C_2 V \qquad 2$$

$$Q_3 = C_3 V$$

The total charge on the system of capacitors is

$$Q = Q_1 + Q_2 + Q_3$$

$$= (C_1 + C_2 + C_3)V \qquad 3$$

The system is therefore equivalent to a single capacitor, of capacitance

$$C = \frac{Q}{V} = C_1 + C_2 + C_3 \qquad 4$$

Thus when capacitors are connected in parallel, their resultant capacitance is the sum of their individual capacitances. It is greater than the greatest individual one.

Series Connection of Capacitors

The figure below shows three capacitors having the right-hand plate of one connected to the left-hand plate of the next, and so on-connected in series.

When a cell is connected across the ends of the system, a charge Q is transferred from the plate H to the plate A, a charge $-Q$ being left on H. This charge induces a charge $+Q$ on plate G; similarly, charges appear on all the other capacitor plates, as in the figure. The potential differences across the individual capacitors are, therefore, given by

$$V_{AB} = \frac{Q}{C_1}$$

$$V_{DF} = \frac{Q}{C_2} \qquad 5$$

$$V_{GH} = \frac{Q}{C_3}$$

The sum of these is equal to the applied potential difference V because the work done in taking unit charge from H to A is the sum of the work done in taking it from H to G, from F to D, and from B to A. Therefore

$$V = V_{AB} + V_{DF} + V_{GH}$$

$$= Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

6

The resultant capacitance of the system is the ratio of the charge stored to the applied potential difference, V . the charge stored is equal to Q, because, if the battery is removed, and the plates HA joined by a wire, a charge Q will pass through that wire, and the whole system will be discharged. The resultant capacitance is therefore given by

$$C = \frac{Q}{V}, \quad \text{or} \quad \frac{1}{C} = \frac{V}{Q} \qquad 7$$

So, by equation (6)

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \qquad 8$$

Thus to find the resultant capacitance of capacitors in series, we must add the reciprocals of their individual capacitances. The resultant is less than the smallest individual.

Note: in parallel connection, the capacitors have the same potential difference, but the charge stored is divided amongst them in direct proportion to the capacitances. In series connection, all the capacitors carry the same charge, which is equal to the charge carried by the system as a whole, Q. The potential difference applied to the system, however, is divided amongst the capacitors in inverse proportion to their capacitances.

Example: Three capacitors C_1, C_2 and C_3 are connected [parallel with one another to give an equivalent capacitance of $1\mu F$, . If the dc supply to the parallel combination is 10 V, the capacitance of C_1 is $0.4\mu F$, and the charge stored by C_2 is $5\mu C$, determine (a) the capacitance of C_2 and of C_3 (b) the energy stored by each capacitor and (c) the total energy stored.

Solution:

Example: Three capacitors C_1, C_2 and C_3 are connected in series with one another. The capacitance of C_1 is $1\mu F$, the voltage across C_2 is 8 V and C_3 stores a charge of $2\mu C$,. If the supply voltage is 20 V dc, calculate the equivalent capacitance of the combination.

Solution:

Series-Parallel Connection of Capacitors

The equivalent capacitance of a complex network of capacitors can be determined by reducing the circuit to basic blocks of series and parallel combinations.

Example: Calculate the equivalent capacitance of a circuit of the type below given that $C_1 = 1\mu F$, $C_2 = 2\mu F$, $C_3 = 3\mu F$, and $C_4 = 4\mu F$,

Solution:

RESISTORS

Resistors are the most common component in electronic circuits. Their main function is to limit current flow or reduce the voltage in a circuit. The resistance may be either fixed or variable. Some fixed resistors are colour coded to indicate their resistance value, while others have their resistance values printed right on the body. The basic unit of resistance is the ohm (Ω). Variable resistors usually have their maximum resistance stamped on them.

RESISTANCE AND RESISTIVITY

The resistance *R* of a conductor is defined as the ratio V/I, where *V* is the potential difference across the conductor and I is the current flowing in it. Thus, if the same potential difference *V* is applied to two conductors *A* and *B*, and a smaller current I flows in *A*, then the resistance of *A* is greater than that of *B*. this can be written as

$$R = \frac{V}{I} \qquad (1)$$

The unit of potential difference *V* is the volt symbol *V*, the unit of current I is the ampere, symbol *A* and the unit of resistance R is the ohm, symbol Ω. The ohm is thus the resistance of a conductor through which a current of one ampere flows when a potential difference of one volt is maintained across it.

From the above equation, it also follows that

$$V = IR \qquad (2)$$

The above relation is known as Ohm's law.

Example: A $5\,\Omega$ resistor is connected across the terminals of a *10V* battery. What is the current in this resistor?

Solution: From the defining equation (1) for *R*, it follows that

$$I = \frac{V}{R}$$

Hence, $$I = \frac{10V}{5\Omega} = 2A$$

The resistance of a conductor is directly proportional to its length ℓ and inversely proportional to its cross-sectional area *A* i.e.

$$R \;\alpha\; \frac{\ell}{A}$$

so that $$R = \rho \frac{\ell}{A} \qquad (3)$$

The constant ρ is an intrinsic material property known as the resistivity of the material. The unit of resistivity is the ohm-metre $(\Omega.m)$.

Resistance to charge flow in a conductor arises because the charge carriers encounter various obstacles that tend to impede their motion. As they collide with these obstacles, the carriers lose momentum and energy gained from the electric field. This loss of energy causes heating of the resistor through which the current flows. The obstacles that scatter the conduction electrons may be impurities dissolved in a pure metal since the attainment of an ideally pure metal is impossible. Even the ideally pure metal crystal (except a superconductor at sufficiently low temperatures) would have a finite resistance because the thermal motion of the atoms in a metal crystal scatters the free electrons. Since the amplitude of this vibrational motion in a crystal increases with increasing temperature, it is to be expected that the scattering of free electrons, and hence the resistivity of a metallic conductor, will also increase as the temperature is raised. That is, indeed, what one observes. The temperature coefficient of resistivity is defined like the coefficient of thermal expansion as

$$\alpha_r = \frac{1}{\rho}\frac{\Delta\rho}{\Delta T} \qquad (4)$$

Values of α_r for several pure metals, a few alloys and carbon, a non-metallic conductor are given above. The resistivity of a semiconductor is much less predictable, depending sensitively not only on temperature but also on the type and concentration of impurities dissolved in the host material.

Example: A 10*m* length of 1*mm* diameter copper wire is wound on a spool. What is the resistance of this length of wire at room temperature?

Solution: From the table, the resistivity of copper at 20^{o}C is $1.69\times10^{-8}\,\Omega.m$. To find *R* we only need to substitute the appropriate numerical values into the equation (3). The area of the wire is

$$A=\frac{\pi d^2}{4}=\frac{\pi}{4}\times10^{-6}\,m^2$$

Thus

$$R=\frac{(10m)(1.69\times10^{-8}\,\Omega.m)}{(\pi/4)\times10^{-6}\,m^2}$$

Example: If the temperature of the wire above increases to *80^{o}C*, what is its resistance at this new temperature?

Solution: We calculate ΔR from equation (4), using the value of α_r from the table. Thus,

$$\Delta R=\alpha_r R\Delta T=(3.9\times10^{-3}\,K^{-1})(0.215\Omega)(60K)=0.050\Omega$$

$$R(80^o C)=R(20^o C)+\Delta R$$

$$=0.215\Omega+0.050\Omega=0.265\Omega$$

CURRENT, VOLTAGE AND POWER DISSIPATION IN A SIMPLE CIRCUIT

The figure below shows the simplest possible direct-current circuit:

It consists of an ideal battery with *emf* ε and an external resistor R (the load). The wires that connect the load to the battery terminals are assumed to be ideal, resistance free conductors. The load may be a light bulb, or any other device that dissipates electrical energy. The direction of current in the external circuit is as indicated, from the positive terminal to the negative. The potential drop across the resistor R is in the direction of current and is given by

$$V = IR \qquad (1)$$

The positive side of R and the positive battery terminal are connected together, and so are the negative side of R and the negative battery terminal. It then follows that

$$\varepsilon = V \tag{2}$$

or

$$\varepsilon = IR \tag{3}$$

for this simple circuit. In the circuit, whenever some charge $\Delta Q = I\Delta t$ is transported from the positive end of the load resistor to the negative end, the charge loses potential energy given by

$$\Delta p.e = V\Delta Q = VI\Delta t \tag{4}$$

Conservation of energy requires that this potential energy appear as energy in some other form: heat, mechanical energy, chemical energy, etc. In each case, the rate at which electrical energy is dissipated in the load is

$$P = \frac{\Delta W}{\Delta t} = VI = I^2 R = \frac{V^2}{R} \tag{5}$$

where the alternative forms are obtained by making use of equation (2). Because Joule used the heat produced by a current passing through a resistor in one of the more precise measurements of the mechanical equivalent of heat, I^2R heating rate in a resistor is often called Joule heating.

At the battery, the charge ΔQ is raised through this same potential difference by the *emf* ε. Hence the battery delivers energy in the amount

$$W = \varepsilon\Delta Q = \varepsilon I\Delta t \tag{6}$$

to the charge ΔQ. The rate at which the battery does work, that is, its power output is therefore

$$P = \varepsilon I \tag{7}$$

From equation (2), we see that the power the battery delivers exactly equals the power dissipation in the load; this must be so since there is no other source or sink of energy in the system.

Example: When a resistor *R* is connected across the terminals of a 10*V* battery, the power dissipated in this resistor is 50 *W*. Find the resistance of the resistor *R* and the current that flows through it.

Solution: From equation (5), we have

$$R = \frac{V^2}{P} = \frac{(10V)^2}{50W} = 2\Omega$$

The current flowing in this resistor is

$$I = \frac{V}{R} = \frac{10V}{2\Omega} = 5A$$

We could also have used the relation (7) and solved for I with the result

$$I = \frac{P}{V} = \frac{50W}{10V} = 5A$$

and then obtain *R* from Ohm's law,

$$R = \frac{V}{I} = \frac{10V}{5A} = 2\Omega$$

RESISTORS IN SERIES AND PARALLEL CONNECTIONS/COMBINATIONS

Every practical electrical circuit is more complicated than the one discussed above. For example, a typical Christmas tree decoration consists of ten or a dozen small lamps placed in series as in figure (a) below or switches connect the battery of an automobile to the starter motor, horn, headlights, heater fan and radio as in figure (b) below. Figures (a) and (b) are examples of series and parallel connections respectively.

The resistors of an electric circuit may be arranged in series, so that the charges carrying the current flow through each in turn (fig c) or they may be arranged in parallel, so that the flow of charge divides between them (d).

Figure (c) shows three passive resistors in series, carrying a current I. If V_{AD} is the potential difference across the whole system, the electrical energy supplied to the system per second is IV_{AD}. This is equal to the electrical energy dissipated per second in all the resistors; therefore

$$IV_{AD} = IV_{AB} + IV_{BC} + IV_{CD}$$

$$V_{AD} = V_{AB} + V_{BC} + V_{CD} \qquad (1)$$

The individual potential differences are given by

$$V_{AB} = IR_1; \qquad V_{BC} = IR_2; \qquad V_{CD} = IR_3$$

Hence,

$$V_{AD} = I(R_1 + R_2 + R_3)$$

Consequently, the effective or total resistance of the system is

$$R = \frac{V_{AD}}{I} = R_1 + R_2 + R_3$$

The physical facts about resistors in series are

Current same through all resistors

Total potential difference = sum of individual potential differences

Individual potential differences directly proportional to individual resistances

Total resistance greater than greatest individual resistance

Total resistance = sum of individual resistances

Example: A chain of twelve identical Christmas tree lights connected in series is to be placed across a 120*V* source of *emf*. Each light should dissipate 15 *W*. What resistance must each of the light bulbs have?

Solution: Since each light must dissipate 15*W*, the total energy delivered by the voltage source must be

$$P = 12 \times 15W = 180W$$

and therefore, the current in the circuit must be

$$I = \frac{P}{\varepsilon} = \frac{180W}{120V} = 1.5A$$

The total potential drop across the twelve bulbs is 120V; since the bulbs are identical, the potential drop across each must be

$$\frac{120V}{12} = 10V$$

Consequently, the resistance of each light bulb is

$$R = \frac{10V}{1.5A} = 6.67\Omega$$

Figure (d) shows three passive resistors connected in parallel between the points *A* and *B*. a current I enters the system at *A* and leaves at *B*, setting up a potential difference V_{AB} between those points. The current branches into I_1, I_2, I_3 through the three elements and

$$I = I_1 + I_2 + I_3$$

But, $I_1 = \frac{V_{AB}}{R_1}$, $I_2 = \frac{V_{AB}}{R_2}$, and $I_3 = \frac{V_{AB}}{R_3}$,

Hence, $$I = V_{AB}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

or

$$\frac{I}{V_{AB}} = \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

where *R* is the effective or total resistance (V_{AB} / I) of the system.

The physical facts about resistors in parallel are:

Potential difference same across each resistor

Total current = sum of individual current

Individual currents inversely proportional to individual resistances

Effective resistance less than least individual resistance

Example: An 800-W toaster, a 200-W lamp and a 300-W electric kettle are connected across 120V as shown below. What are the resistances of the individual components and what is the equivalent resistance of this combination?

Solution: Each component is connected across 120V when it dissipates the stated power. Hence, the resistances can be obtained as

$$R = \frac{V^2}{P}$$

or $$R_t = \frac{(120V)^2}{800W} = 18\Omega$$

$$R_L = \frac{(120V)^2}{200W} = 72\Omega$$

$$R_k = \frac{(120V)^2}{300W} = 48\Omega$$

The equivalent resistance of the combination is

$$R_{eff} = \left[\frac{1}{18\Omega} + \frac{1}{72\Omega} + \frac{1}{48\Omega}\right]^{-1} = 11.1\Omega$$

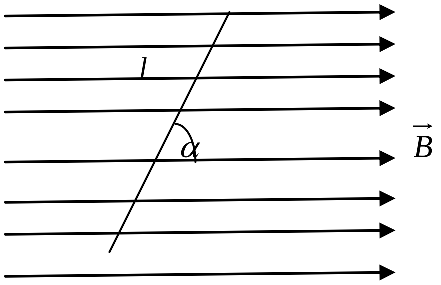
MAGNETIC FIELD

The region around a magnet, where a magnetic force can be experienced is known as a magnetic field. The direction of the field is taken as the direction of the force on a north pole if placed in the field. The magnetic field due to a solenoid will depend on the current flowing in it. If this current can be varied by adjusting the rheostat, then the larger the current in the solenoid, the larger is the force i.e. a larger current produces a stronger magnetic field. Magnetic field is represented by a vector quantity that is given the symbol $\vec{B}$. This is called the flux density or magnetic induction.

The magnitude of the force $\vec{F}$ is given by

$$\left|\vec{F}\right| = BIl\sin\alpha$$

where α is the angle between the magnetic field and the conductor as in the figure below:



The SI unit of B is the Tesla(T) where 1 T = 1 Weber per metre squared (Wbm^{-2})
One Tesla is the flux density of a uniform field, when the force on a conductor **1** meter long placed perpendicular to the field and carrying a current of **1** ampere is **1** Newton.
The unit of B is clearly $NA^{-1}m^{-1}$ which turns out to be a $kgC^{-1}s^{-1}$. We call this unit the Tesla, although the older name for this unit, the Weber per square meter is still used frequently. Another unit in common usage, the gauss (G) is defined by

$$1T = 10^4 G$$

Since the gauss is not an SI unit, we should always convert it to Teslas before using it in our equations.

NOTE: The north pole of a magnet is defined to be the pole that points north when the magnet is freely suspended.

Example: A wire carrying a current of 10A and 2m in length is placed in a field of flux density 0.15T. Determine the force on the wire if it is placed

(a) At right angles to the field

(b) At 45^o to the field

(c) Along the field

Solution: From the equation $|\vec{F}| = BIl\sin\alpha$, we have

(a) $F = 0.15T \times 10A \times 2m \times \sin 90 = 3N$

(b) $F = 0.15T \times 10A \times 2m \times \sin 45 = 2.12N$

(c) $F = 0.15T \times 10A \times 2m \times \sin 0 = 0N$

RADIACTIVITY

Half-life and Decay constant

Radioactivity is the emission of an alpha or beta particles and gamma rays due to the disintegrating nuclei of atoms and the disintegrations obey the statistical law of chance. Although we cannot tell which particular atom is likely to disintegrate next, the number of atoms disintegrating per second $\frac{dN}{dt}$ is directly proportional to the number of atoms N present at that instant. Hence,

$$\frac{dN}{dt} \alpha N$$

or

$$\frac{dN}{dt} = -\lambda N$$

where λ is a constant characteristic of the atom concerned called the radioactive decay constant. The negative sign indicates that N becomes smaller when t increases. If N_o is the number of radioactive atoms present at a time $t = 0$, and N is the number at the end of a time t, then by integration, we have

$$N = N_o e^{-\lambda t}$$

This shows that the number N of undecayed atoms left decreases exponentially with the time t.
The half-life $T_{1/2}$ of a radioactive element is defined as the time taken for half the atoms to disintegrate i.e. in a time $T_{1/2}$ the radioactivity of the element diminishes to half its value. Hence,

$$\frac{N_o}{2} = N_o e^{-\lambda t}$$

or

$$\frac{1}{2} \equiv 2^{-1} = e^{-\lambda T_{\frac{1}{2}}}$$

Taking logarithms to base e on both sides of the equation and simplifying yields

$$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

Various radioactive elements have got different half-lives. For example, Uranium I (238) has half-life of the order 4500 million years; Radium has one of 1600 years, Radium F of about 138 days, Radium B of about 27 minutes and Radium C about 10^{-4} seconds. To find the unit for measuring the radioactivity of a sample, we define a quantity called the activity which is the number of decays in the sample per unit time ($-\frac{dN}{dt}$). Its SI unit is the Becquerel (*Bq*-number of decays per second). Another unit for activity is the curie (*Ci*) where

$$1C_i = 3.7 \times 10^{10} Bq$$

From equations (1) and (2), it follows that the activity of a radioactive sample is

$$\text{Activity } \lambda N = \frac{N \ln 2}{T_{1/2}}$$

Example: Find the activity of *1g* of strontium-90 source if the half-life is 28 years.

Solution:

Types of emissions (radiations) from naturally radioactive Nuclei and their characteristics

Alpha Particles

They have a limited range in air i.e. the count rate decreases as the distance between the source and detector increases.

They can be stopped by a thin piece of paper.

They have positive charge (+2e).

They undergo a small amount of deflection by a magnetic field.

They ionize the air.

Alpha particle is a helium nucleus i.e. helium atom which has lost two electrons

Beta Particles

Are electrons moving at high speed.

They have a greater penetrating power of materials than alpha particles (can be stopped by lead).

They have a greater range in air than alpha particles since their ionization of air is relatively smaller.

They are strongly deflected by a magnetic field and the direction of deflection is opposite to the deflection of alpha particles in the same field.

Gamma rays

They can penetrate large thicknesses of metals, but they have far less ionizing power in gases than beta particles.

They carry no charge.

They are not deflected by magnetic field. This is consistent with the fact that gamma rays are electromagnetic waves that carry no charge.

Since gamma rays are electromagnetic waves, their intensity varies inversely as the square of the distance between the source and the detector i.e.

$$I \alpha \frac{1}{r^2}$$

They are electromagnetic waves that travel through the vacuum with the speed of light,

They carry energy given by

$$E = hf$$

NUCLEAR EMISSIONS AND STABILITY

Since unstable nuclei are radioactive, their decay may occur in three main ways:

Alpha Particle emission (α): If the nucleus has excess protons, an α emission would reduce the protons by two (2) and the neutrons by two (2). If A is the nucleon (mass) number and Z is the proton number (atomic) number of atom X, then

$${}_{Z}^{A}X \rightarrow {}_{Z-2}^{A-4}Y + {}_{2}^{4}He$$

Beta Particle emission (β^-): If the nucleus has too many neutrons for stability, then the neutron-proton ration is reduced by β^- particle (electron) emission. The neutron changes to a proton so that

$${}_{0}^{1}n \rightarrow {}_{1}^{1}H + {}_{-1}^{0}e$$

Here, A remains unchanged but Z increases by one (1).

Beta-Plus Particle or positron emission (β^+): If the nucleus is deficient in neutrons, a decay by beta-plus β^+, emission may occur. A Proton changes to a neutron so that

$$ {}_1^1H \rightarrow {}_0^1n + {}_1^0e $$

Here A is unchanged but Z decreases by one (1).

Example: A uranium-238 nucleus decays by emission of an alpha particle according to the following scheme

$$ {}_{92}^{238}U \rightarrow {}_{90}^{234}Th + {}_2^4He $$

where the daughter nucleus is thorium. The very long half-life of this decay explains why there is still some Uranium-238 left on earth. In addition, it acts as a continual source for this isotope of thorium. The thorium formed in this decay is also radioactive. It decays by beta emission according to the equation

$$ {}_{90}^{234}Th \rightarrow {}_{91}^{234}Pa + {}_{-1}^{0}e $$

where the daughter nucleus is protactinium (why does the atomic number increase in this decay? Why doesn't the mass number change?).

The Protactinium in turn decays to Uranium-234 so that

$$ {}_{91}^{234}Pa \rightarrow {}_{-1}^{0}e + {}_{92}^{234}U $$

Several other steps occur in this radioactive series before the final stable element of the series is reached i.e. an isotope of lead ${}_{82}^{206}Pb$

ISOTOPE

When the mass spectrometer is used to measure nuclear masses, an interesting effect is observed. Very frequently, one finds that a certain element will give rise to two or more different beams in the spectrometer i.e. particles will appear at the detector at two or more very well defined radii, an indication that nuclei of the same element may have different masses

Nuclei having the same charge but different masses are called isotopes of the element i.e. they have the same number of protons but a different number of neutrons. In order to classify the nuclei in terms of their mass, charge and nucleon number, it is customary to designate an element whose symbol is *X* as where A is the mass number of the nucleus and *Z* is the atomic number. For example, the chlorine isotopes would be represented as ${}_{17}^{35}Cl$ and ${}_{17}^{37}Cl$; both isotopes have the same atomic number *Z* = 17, but one has a mass number *A* = 35 and the other has *A* = 37.

NOTE: *Z* is the number of protons in the nucleus and *A* is the number of nucleons (protons + neutrons). In the period table, atomic masses given are the average value of the isotopic masses found in nature. For example, when chemically pure neon is sent through the mass spectrometer, it appears to consist of three different types of nuclei:

Species 1	Mass 20	Relative abundance = 90.9
Species 2	Mass 21	Relative abundance = 0.3
Species 3	Mass 22	Relative abundance = 8.8

The average mass of the three isotopes of Neon is

$$ M_{av} = 20\times\frac{90.9}{100} + 21\times\frac{0.3}{100} + 22\times\frac{8.8}{100} = 20.746 $$

The atomic masses of many isotopes are the masses of the nuclei plus the atomic electrons. These masses are given in atomic mass unit *u*.

Example: What fraction of the atomic mass of Uranium-235 is due to its electrons?

Solution: Since the atomic number of uranium is 92, it will also have 92 electrons. Given that the mass of the electron is 9.1×10^{-31} or $0.00055u$, then the fraction of the mass due to electrons is

$$\frac{92 \times 0.00055u}{235u} = 2.15 \times 10^{-4}$$

Therefore, for many purposes, the mass of the electrons can be ignored.

TUTORIAL: At a certain instant, a piece of radioactive material contains 10^{12} atoms. The half-life of the material is 30 days.

(i) Calculate the number of disintegrations in the first second.
(ii) How long will it take for 10^4 atoms remain?
(iii) What is the count rate at this time?

Solution:

HEAT AND THERMODYNAMICS

TEMPERATURE

Temperature is commonly associated with how hot or cold an object feels when we touch it. Understanding the concept of temperature requires the understanding of *thermal contact* and *thermal equilibrium*. Two objects are in **thermal contact** if energy can be exchanged between them while, two objects are in **thermal equilibrium** if they are in thermal contact and there is no net exchange of energy. The exchange of energy between two objects because of differences in their temperatures is called **heat.** Using these ideas, a formal definition of temperature can be developed i.e. temperature is the property that determines whether or not an object is in thermal equilibrium with another i.e., **Two objects in thermal equilibrium with each other are at the same temperature**. It also corresponds to primary sensations of hotness and coldness. These sensations are not reliable enough for scientific work because they depend on contrast. A thermometer such as the familiar mercury-in-glass instrument is a device whose readings depend on hotness or coldness of the system that it is in contact with and which we consider more reliable because different thermometers of the same type agree with one another better than different people do.

Types of Thermometers

The temperature of a system is not a fixed number but depends on the type of thermometer used and on the temperature scale adopted. In general, thermometers use some measurable property of a substance which is sensitive to temperature changes. For example, the constant-volume gas thermometer uses the pressure change with temperature of a gas at constant volume, the resistance thermometer uses the change of electrical resistance of a pure metal with temperature, the mercury-in-glass thermometer depends on the change in volume of mercury with temperature relative to that of glass and a thermoelectric thermometer depends on the electromotive force change with temperature of two metals joined together.

Thermodynamic Temperature Scale

This is the standard temperature scale adopted for scientific measurements. It is denoted by T and is measured in Kelvin (K). It uses one fixed point, the triple-point of water, which is the temperature at which saturated water-vapour, pure water and melting ice are all in equilibrium. The triple-point temperature is defined as 273.16 K. Using the constant-volume gas thermometer, for example, the gas pressure P_{tr} is measured at the triple-point as 273.16K. If the pressure is P at an unknown temperature T on the thermodynamic scale, then by definition,

$$T = \frac{P}{P_{tr}} \times 273.16K$$

With a platinum resistance thermometer, the resistance R can also be measured at an unknown temperature. If R_{tr} is the resistance at the triple-point, then the temperature T_{pt} on the thermodynamic scale is given by

$$T_{pt} = \frac{R}{R_{tr}} \times 273.16K$$

Celsius Temperature Scale (formerly known as centigrade scale)

The Celsius temperature, symbol T_C, is defined by

$$T_C = T - 273.15$$

where T is the thermodynamic temperature. In this scale, the ice point is 0^o C and the steam point is 100^oC. The temperature change or interval of one degree (1^oC) is exactly the same as the temperature interval 1 K on the thermodynamic scale. We notice that, whereas the triple-point has numerical value of 273.16 K, the ice point has a temperature of 273.15 K. the slight difference is due to the difference in pressure (4.6 *mmHg* at the triple point and 760 *mmHg* at the ice point) and to the removal of dissolved air from the distilled water used for the triple point.

The Fahrenheit scale employs a smaller degree scale than the Celsius scale and a different zero of temperature. The relation between the Celsius and Fahrenheit scales is

$$T_F = \frac{9}{5} T_C + 32$$

where T_F is Fahrenheit temperature and T_C is Celsius temperature. For example, 0^oC on the Celsius scale measures the same temperature as 32^oF on the Fahrenheit scale, whereas 5^oC on the Celsius scale is equivalent to 41^oF on the Fahrenheit scale.

Internal energy (*U*): is the energy associated with the microscopic components of a system (the atoms and molecules of the system). The internal energy includes kinetic and potential energy associated with the random, translational, rotational, and vibrational motion of the particles that make up the system, and any potential energy bonding the particles together. The higher the temperature of the gas, the greater the kinetic energy of the atoms and, hence, the internal energy of the gas.

Heat

This is a form of energy that can be transferred between a system and its environment due to a temperature difference between them. The symbol *Q* is used to represent the amount of energy transferred by heat between a system and its environment. The unit of heat is the **calorie**, defined as **the energy necessary to raise the temperature of 1 g of water by one degree**. We are interested in heat because it is the commonest form of energy and because changes of temperature have great effects on our personal comfort and on the properties of substances such as water, which we use daily.

Heat Capacity and Specific Heat Capacity

The heat capacity of a substance is the quantity of heat required to raise its temperature by one degree. It is expressed in Joules per Kelvin (JK^{-1}). Quantitatively, is given by

$$C = \frac{Q}{\Delta T}$$

On the other hand, the Specific heat capacity c_s of a substance is the heat required to raise the temperature of one kilogramme of it by one degree. It is the heat capacity per unit mass of a substance and is expressed in Joules per Kelvin per kilogram ($Jkg^{-1}K^{-1}$). If a quantity of energy *Q* is transferred to a substance of mass *m*, changing its temperature by

$$\Delta T = T_f - T_i$$

then the **specific heat capacity** c_s of the substance is defined by

$$c_s = \frac{Q}{m\Delta T}$$

The energy required to raise the temperature of 0.500 kg of water by 3.00°C, for example, is

$$Q = mc_s\Delta T = 0.5kg \times 4186Jkg^{-1}K^{-1}3K = 6279J$$

Note that when the temperature increases, ΔT and *Q* are *positive*, corresponding to energy flowing *into* the system. When the temperature decreases, ΔT and *Q* are *negative*, and energy flows *out* of the system.

Water has the highest specific heat relative to most other common substances. This high specific heat is responsible for the moderate temperatures found in regions near large bodies of water. As the temperature of a body of water decreases during winter, the water transfers energy to the air, which carries the energy landward when prevailing winds are toward the land, keeping the coastal areas much warmer than they would otherwise be. Winters are generally colder in the some coastal areas, because the prevailing winds tend to carry the energy away from land.

The fact that the specific heat of water is higher than the specific heat of sand is responsible for the pattern of airflow at a beach. During the day, the Sun adds roughly equal amounts of energy to the beach and the water, but the lower specific heat of sand causes the beach to reach a higher temperature than the water. As a result, the air above the land reaches a higher temperature than the air above the water. The denser cold air pushes the less dense hot air upward (due to Archimedes's principle), resulting in a breeze from ocean to land during the day. Because the hot air gradually cools as it rises, it subsequently sinks, setting up the circulation pattern.

From the definition of specific heat capacity, it follows that, for a particular object,

Heat capacity C = Mass (m) $\times$ specific heat capacity (c_s)

The specific heat capacity c_s of copper, for example, is about $400Jkg^{-1}K^{-1}$. Hence, the heat capacity of 5kg of copper is

$$C = mc_s = 5kg \times 400Jkg^{-1}K^{-1} = 2000JK^{-1}$$

If the temperature of the copper rises by 10^oC, then the heat gained is

$$Q = mc_s\Delta T = 5kg \times 400Jkg^{-1}K^{-1} \times 10K = 20kJ$$

Generally, the heat gained or lost by an object is given by

$$Q = mc_s\Delta T$$

where m is the mass of the object, c_s its specific heat capacity and ΔT its temperature change.

From $Q = mc_s\Delta T$ the temperature change ΔT of an object of mass m which losses or gains quantity of heat Q is

$$\Delta T = \frac{Q}{mc_s}$$

So a given loss of heat Q to the surroundings, the temperature fall ΔT of a small mass of a substance in a room is greater than a large mass of the substance at the same temperature. The rate at which a hot object (solid or liquid) cools depends on the nature and area of its surface, in addition to its temperature, mass and specific heat capacity.

MEASUREMENT OF SPECIFIC HEAT CAPACITY

Electrical Method.

Suppose in an experiment, the voltmeter reads $12V$, the ammeter reads $4.0A$ and the temperature of the block of mass $1.0kg$ rises by 16^oC in 56 minutes. Then,

Heat (electrical energy) supplied = Power $\times$ Time

i.e.

$$Q = Pt = IVt = 4.0A \times 12V \times 300s = 14400J$$

If c_s is the specific heat capacity of the metal, then assuming negligible heat losses,

$$Q = mc_s\Delta T$$

So that

$$c_s = \frac{Q}{m\Delta T} = \frac{14400J}{1kg \times 16K} = 900Jkg^{-1}K^{-1}$$

The electrical energy supplied is

$$\Delta Q = IVt$$

and from the principle of conservation of energy, this is equal to the rise in internal energy $\Delta U (= mc_s\Delta T)$ of the metal plus the heat losses h by cooling plus the external work ΔW done against external atmospheric pressure by the metal when it expands on warming. Since metals expand very slightly in volume on warming, ΔW can be neglected in an experiment. So,

$$IVt = mc_s\Delta T + h$$

If ΔT is not corrected for heat losses, then from

$$IVt = mc_s\Delta T$$

the result for c_s is too high.

Method of Mixtures

A common method of measuring the specific heat capacities of solids and liquids is the Method of Mixtures. As an illustration, suppose a metal of mass 0.2*kg* at 100^oC is lowered into 0.08*kg* of water at 15^oC contained in a calorimeter of mass 0.12*kg* and specific heat capacity $400Jkg^{-1}K^{-1}$. The final temperature reached is 35^oC. Then assuming negligible heat losses and taking the specific heat capacity of water as $4200Jkg^{-1}K^{-1}$, then

Heat capacity of calorimeter, $C_c = 0.12kg \times 400Jkg^{-1}K^{-1} = 48JK^{-1}$

Heat capacity of water $C_w = 0.08kg \times 4200Jkg^{-1}K^{-1} = 336JK^{-1}$

Heat gained by water + calorimeter = $(336 + 48) \times (35\text{-}15)J$

Heat lost by hot metal = $0.2\text{kg} \times c_s \times (100\text{-}35)J$

From the energy conservation law,

Heat lost by metal = heat gained by water + calorimeter

$$0.2 \times c \times 65 = 384 \times 20$$

Therefore,

$$c = \frac{384 \times 20}{0.2 \times 65} = 590 Jkg^{-1}K^{-1}$$

Specific Latent heat of Evaporation

is the heat required to convert a unit mass of a liquid at its boiling point into vapour at the same temperature. Its SI unit is Jkg-1. Quantitatively, it is given by

Heat given by steam condensing + Heat given by condensed water cooling from 100°C to T_2 = Heat taken by calorimeter and water

$$ml_{ev} + mc_w(100 - T_2) = (m_1c_w + C)(T_2 - T_1)$$

where m_1 is the mass of water in the calorimeter, c_w is the specific heat capacity of water and C is the heat capacity of the metal calorimeter. Hence,

$$l_{ev} = \frac{(m_1c_w + C)(T_2 - T_1)}{m} - c_w(100 - T_2)$$

where T_1 and T_2 are the initial and final temperatures of the water.

Specific Latent heat of Fusion of a Solid

is the heat required to convert unit mass of it at its melting point into a liquid at the same temperature. It is also expressed in Jkg^{-1}. For ice, it is given by

Heat given by calorimeter + water in cooling = heat used in melting + heat used in warming melted ice from 0^oC to T_2

$$(m_1c_w + C)(T_1 - T_2) = ml_F + mc_w(T_2 - 0)$$

where m_1 is mass of calorimeter, c_w is specific heat capacity of water, C(= m_cc_c) is the heat capacity of calorimeter and T_1 is the initial temperature. Therefore,

$$l_F = \frac{(m_1c_w + C)(T_1 - T_2)}{m} - c_wT_2$$

Example: An electric kettle with a 2.0kW heating element has a heat capacity of $400Jk^{-1}$. 1.0 *kg* of water at 20^oC is placed in the kettle. It is switched on and it is found that 13 minutes later the mass of water in it is 0.5*kg*. Ignoring heat losses, calculate a value for the specific latent heat of evaporation of water.

Solution: Total heat supplied $Q = Pt = 2000kW \times 13 \times 60s = 1.56 \times 10^6 J$

Heat used for kettle ($Q_{ket} = C\Delta T$) = 400$Jkg^{-1}K^{-1}\times$(100-20)K = 0.032$\times 10^6$J

Heat used to raise temperature of water from 20^{o}C to 100^{o}C ($= mc_w\Delta T$)

= 1$kg\times$4200$Jkg^{-1}K^{-1}\times$80K

= 0.336$\times 10^6$J

Total heat to change water at 100^{o}C to steam at 100^{o}C (= *ml)*

= 1.56$\times 10^6 J$ – (0.032 + 0.336) $\times 10^6$J

= 1.192$\times 10^6 J$

Since mass of water changed to steam is (1 – 0.5) *kg* = 0.5*kg*

then,

$$l = \frac{1.192\times 10^6\,J}{0.5kg} = 2.38\times 10^6\,Jkg^{-1}$$

THERMAL EXPANSION OF SOLIDS, LIQUIDS AND GASES

SOLIDS: LINEAR EXPANSION

Most solids increase in length when they are warmed. Suppose we measure the length l_1 of a metal rod at room temperature T_1 and then measure the expansion *e* of the rod at a higher temperature T_2. The increase in length λ of unit length of the material for one degree temperature rise is then given by

$$\lambda = \frac{e}{l_1(T_2 - T_1)} = \frac{l_2 - l_1}{l_1(T_2 - T_1)}$$

The quantity λ is called the mean linear expansivity of the metal over the range T_1 to T_2 and has the unit K^{-1} in *SI* units. Its dimensions are

$$[\lambda] = \frac{[Length]}{[Length]\times[Temperature]} = [Temperature]^{-1}$$

From the definition of λ, we can estimate the new length of the rod λ at a temperature T_2 as

$$l_2 = l_1\{1 + \lambda(T_2 - T_1)\}$$

where l_1 is the length of the rod at temperature T_1.

Example: The metal pendulum of a metal clock has a linear expansivity of $2\times 10^{-5}K^{-1}$. If the period is 2*s* at 15^o*C*, calculate the loss or gain in 10 hours when the temperature rises to 25^o*C*.

Solution: Owing to temperature rise, the length of the pendulum increases from l_1 at 15°C (period T_1) to l_2 at 25°C (period T_2). Now, since

$$\text{Period } \alpha \sqrt{Length}$$

then

$$\frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}}$$

The new length l_2 is

$$l_2 = l_1(1 + \lambda(T_2 - T_1))$$

$$= l_1(1 + 10\lambda)$$

since the temperature rise is 10K. Hence,

$$\frac{l_2}{l_1} = 1 + 10\lambda = 1 + 10 \times 2 \times 10^{-5} = 1.0002$$

Therefore,

$$\frac{T_2}{T_1} = \frac{T_2}{2s} = \sqrt{1.0002} = 1.0001$$

or

$$T_2 = 2.0002s$$

The clock therefore records 2s when the time change is actually 2.0002s, that is, the clock loses 0.0002s in 2.0002s. so in 10 hours, the clock looses

$$\text{Lose in 10 hours } = \frac{10hrs \times 3600s \times 0.0002s}{2s \times 1hrs} = 3.6s$$

LIQUIDS: Cubic Expansivity

The temperature of a liquid determines its volume, but its vessel determines its shape. Consequently, the only expansivity which we can define for a liquid is its cubic expansivity γ. Since most liquids, like most solids, do not expand uniformly, γ is not constant over a wide range of temperature. Therefore, over a given range T_1 to T_2, the mean cubic expansivity γ is defined as

$$\gamma = \frac{V_2 - V_1}{V_1(T_2 - T_1)}$$

where V_1 and V_2 are the volumes of a given mass of liquid at the temperatures T_1 and T_2.

Below are the mean expansivities of water and mercury at various temperatures:

WATER **Temperature** (^oC)	$\gamma \times 10^{-4} K^{-1}$
5-10	0.53
10-20	1.50
20-40	3.02
40-60	4.58
60-80	5.87
MERCURY **Temperature** (^oC)	$\gamma \times 10^{-4} K^{-1}$
< 30	1.81
<100	1.82
<300	1.87

NOTE: If V_1 and V_2 are the volumes of a unit mass of the liquid at T_1 and T_2, then

$$V_2 = V_1\{1 + \gamma(T_2 - T_1)\}$$

The densities of the liquid at the two temperatures are

$$\rho_1 = \frac{1}{V_1} \text{ and } \rho_2 = \frac{1}{V_2}$$

Hence,

$$\frac{1}{\rho_2} = \frac{1}{\rho_1}\{1 + \gamma(T_2 - T_1)\}$$

or

$$\rho_2 = \frac{\rho_1}{1 + \gamma(T_2 - T_1)}$$

EXPANSION OF GASES

The rate at which the volume of a gas increases with temperature can be defined by a quantity known as Gas Expansivity at constant pressure α_p or volume:

$$\alpha_p = \frac{Volume \text{ at } T_2{}^oC - Volume \text{ at } 0^oC}{Volume \text{ at } 0^oC} \times \frac{1}{\Delta T}$$

GAS LAWS

Here, we are concerned with the relationship between the temperature, pressure and volume of a gas. This can be expressed in very simple laws which, then, reduce to a simple equation known as the equation of state.

Pressure and Volume: Boyle's Law

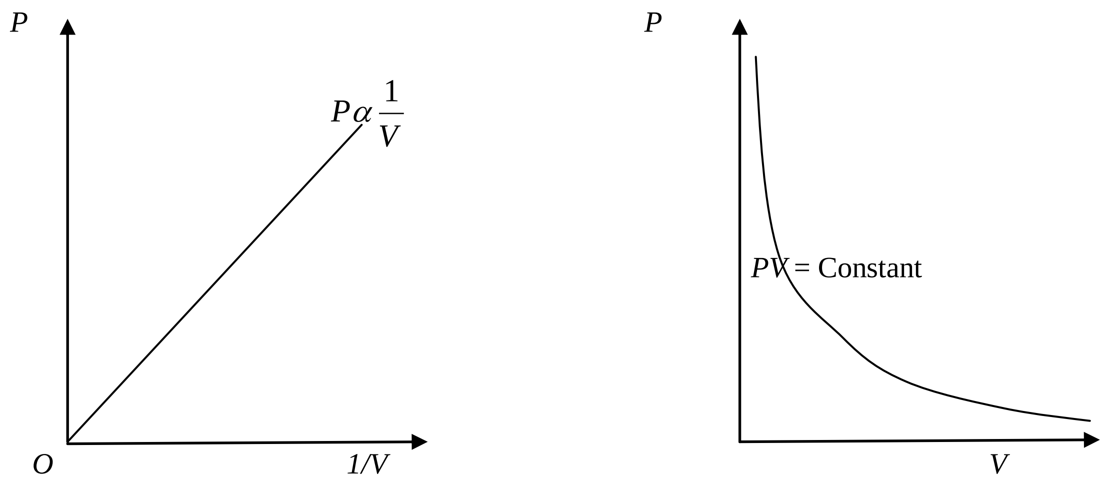
The law states that: the pressure of a given mass of a gas at constant temperature is inversely proportional to its volume i.e.

$$P \alpha \frac{1}{V}$$

so that

$$PV = cons\tan t$$

Graphically, the law can be demonstrated as shown below:



Example: A fault barometer tube has some air at the top above the mercury. When the length of the air column is 250*mm*, the reading of the mercury above the outside level is 750*mm*. when the length of the air column is decreased to 200*mm*, the reading of the mercury above the outside level becomes 746*mm*. Calculate the atmospheric pressure.

Solution: Since the tube has a uniform cross-section, then the volume is proportional to the length so that;

Initial volume V_1 is proportional to the length 250*mm*

Initial air pressure is $P_1 = (A - 750mmHg)$

Also, new volume V_2 is proportional to 200*mm* and

New pressure of air is $P_2 = (A - 746mmHg)$

where *A* is the atmospheric pressure. From Boyle's law,

$$P_1V_1 = P_2V_2$$

so that

$$250mm \times (A - 750mmHg) = 200mm \times (A - 746mmHg)$$

or

$$5(A - 750mmHg) = 4(A - 746mmHg)$$

Therefore,

$$A = 766mmHg$$