Solution to Economic Load Dispatch using Particle Swarm Optimization

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Abstract—This paper proposes to determine the feasible optimal solution of the economic load dispatch power systems problem using Particle Swarm Optimization (PSO) considering various generator constraints. The objective of the proposed method is to determine the steady-state operating point which minimizes the fuel cost, while maintaining an acceptable system performance in terms of limits on generator power, line flow, prohibited operating zone and non-linear cost function. Three different inertia weights; a constant inertia weight CIW, a time-varying inertia weight TVIW, and global-local best inertia weight GLbestIW, are considered with the (PSO) algorithm to analyze the impact of inertia weight on the performance of PSO algorithm. The PSO algorithm is simulated for each of the method individually. It is observed that the PSO algorithm with the proposed inertia weight (GLbestIW) yields better results, both in terms of optimal solution and faster convergence.

Keywords: Classical particle swarm optimization (CPSO), Economic load dispatch.

I. INTRODUCTION

Economic load dispatch (ELD) is a non-linear constraint based optimization problem in power systems that have the objective of dividing the total power unit demand among the online participating generators economically while satisfying the essential constraints [1]. The goal is to optimize a selected objective function such as fuel cost via optimal adjustment of the power system control variables, while at the same time satisfying various inequality and equality constraints. Equality constraints are power flow equations, inequality constraints set limits on the control variables and the operating limits of the power system dependant variables. The goal is to find values of the variables that minimize or maximize the objective function while satisfying the constraints. PSO is a population based optimization strategy, particularly well suited for stochastically finding extrema in continuous non-linear functions [2]. The approach is derived in part from the way flocks of birds and swarms in nature search for food. A “swarm” is an apparently disorganized collection (population) of moving individuals that tend to cluster together while each individual seems to be moving in a random direction [3 - 4]. In PSO, a set of particles sample a search space and then adjust their search directions to sample near to their fitter neighbours. The set of neighbour connections between all of the particles forms the swarms topology or sociometry [3 - 5] and affects the swarm’s exploitation and exploration behaviour [3]. There have been two basic topologies used in the literature Ring Topology (neighbourhood of 3) and Star Topology (global neighbourhood). Practically, the real world input-output characteristics of the generating units are highly nonlinear, non-smooth and discrete in nature owing to prohibited operating zones, ramp rate limits and multi-fuel effects.

Particle swarm optimization (PSO) algorithm is proposed to solve the various types of economic load dispatch problems in power systems. The feasibility of the proposed method is demonstrated on six different systems and the numerical results were compared with other evolutionary computing techniques [6 - 26].

II. PROBLEM FORMULATION

The objective of the economic load dispatch problem is to initialize the total fuel cost

\[
\text{Min } F_T = \sum_{i=1}^{N} F_i \quad \text{or} \quad \min F_T = \sum_{i=1}^{N} F_i(P_D) \quad (i)
\]

Subject to constraints:

\[
P_D + P_L = \sum_{i=1}^{N} P_i \quad (ii)
\]

Where \(F_T\) total production cost (KShs/hr); \(F_i(P_D)\) is incremental fuel cost function (KShs/hr); \(N\) is number of generating units;

\(P_D\): Total real power unit demand (MW)

\(P_L\) : Total power losses (MW)

OPERATING COST OF A THERMAL POWER PLANT:

The factors influencing power generation are operating efficiencies of generators, fuel cost and transmission losses. The total cost of generation is a function of the individual generation of the sources which can take values within certain constraints. The problem is to determine the generation of different plants such that total operating cost is minimum. The input to the thermal plant is generally measured in Btu/hr and the output power is the active power in MW. A simplified input-output curve of a thermal unit known as heat-rate curve:
The fuel cost functions of the generating units are generally characterized by second-order polynomials as:
\[ F_{P_i} = a_i + b_i P_i + c_i P_i^2 \]
where
\( P_i \) is the real output power generation of the \( i \)th unit.

Steam input-output equation + Ripple-like heat rate curve
\[ \min F_{P_i} = \sum_{i=1}^{N} (a_i + b_i P_i + c_i P_i^2 + |e_i| \sin(f_i (P_i - P_{i[\text{min}]}) )) \]

(iii) The incremental fuel-cost curve is a measure of how costly it will be to produce the next increment of power.
\[ \text{dCi/dPi} = 2 \]

Calculation of Input-Output characteristic parameters:
The parameters of the input-output characteristic of any generating unit can be determined by the following approaches:
1. Based on the experiments of the generating unit efficiency.
2. Based on the historic records of the generating unit operation.
3. Based on the design data of the generating unit provided by the manufacturer.

In the practical power systems, we can easily obtain the fuel statistic data and power output statistic data. Through analyzing and computing data set (Fk, Pk), we can determine the shape of the input-output characteristic and the corresponding parameters.

2.4 SYSTEM CONSTRAINTS:
Generally there are two types of constraints [30]
1) Equality constraints
2) Inequality constraints

For i = 1 ... N
\[ \text{b) real power balance:} \quad \sum_{i=1}^{N} P_i = P_{\text{Load}} + P_D \]
\[ \text{Power loss:} \quad P_{\text{Loss}} = \sum_{m=1}^{N} \sum_{n=1}^{N} P_m P_{mn} P_n \]

and \( P_n \) are the real power injections at the \( n \)th and \( m \)th buses in the network. \( B_{mn} \) are the B-coefficients of transmission loss formula.

b) real power generation limit: \( P_{\text{min}} \leq P_i \leq P_{\text{max}} \)
For \( i = 1 \ldots N \)
\[ \text{c) reactive power generation limit:} \quad Q_{\text{min}} \leq Q_i \leq Q_{\text{max}} \]
\( B_{\text{max}} \) is transmission loss coefficients; \( P_{\text{max}} \) is the minimum limit of the real power of the \( i \)th unit (MW); \( P_{\text{max}} \) is maximum limit of the real power of the \( i \)th unit (MW).

INEQUALITY CONSTRAINTS:
1) Generator Constraints: The KVA loading of a generator can be represented as \( \sqrt{P^2 + Q^2} \). The KVA loading should not exceed a pre-specified value to limit the temperature rise.

The swarm can be represented by a \( D \)-dimensional vector,
\[ X_i = (x_{i1}, x_{i2}, \ldots, x_{iD}) \]
\[ \text{The velocity (position change) of this particle, can be represented by another D-dimensional vector} \]
\[ V_i = (v_{i1}, v_{i2}, \ldots, v_{iD}) \]

The best previously visited position of the \( i \)th particle is denoted as \( P_i = (p_{i1}, p_{i2}, \ldots, p_{iD}) \). Defining \( g \) as the index of the best particle in the swarm (i.e., the g-th particle is the best), and let the superscripts denote the iteration number, then the swarm is manipulated according to the following two equations (Eberhart et al., 1996) [1 - 3]:

Initial version of PSO – no actual mechanism for controlling the velocity of a particle, maximum value \( V_{\text{max}} \) was imposed on it:
\[ v_{n+1} = v_n + \alpha \phi (p_{g(n)} - x_n) + \beta \phi (p_{n}(n) - x_n) \]
\[ x_{n+1} = x_n + v_{n+1} \]

III. VELOCITY \( V_{\text{MAX}} \):
- An important parameter in PSO; typically the only one adjusted.
- Clamps particles velocities on each dimension.
- Determines ‘fitnessness’ with which regions are searched.
- Large values of \( V_{\text{max}} \) could result in particles moving past optimal solutions.
- Small values could result in insufficient exploration of the search space.

This lack of a control mechanism for the velocity resulted in low efficiency for PSO, compared to EC techniques (Angeline, 1998). Specifically, PSO located the area of the optimum faster than EC techniques, but once in the region of the optimum, it could not adjust its velocity stepsize to continue the search at a finer grain.

The aforementioned problem was addressed by incorporating a weight for the previous velocity of the particle. Thus in the largest versions of PSO, equations (xi) and (xii) are changed to the following ones:
\[ v_{n+1} = \alpha \phi (p_{g(n)} - x_n) + \beta \phi (p_{n}(n) - x_n) \]
\[ x_{n+1} = x_n + v_{n+1} \]

\( w = \text{inertia weight} \)
\( c_1, c_2 = \text{two positive constants; cognitive and social parameter respectively} \)
K = constriction factor which is used, alternatively to \( w \) to limit velocity.
(Eberhart and Shi, 1998; Shi and Eberhart, 1998a, Shi and Eberhart, 1998b) [1 - 3]

IV. PSO PARAMETERS
The role of inertia weight, \( (w=1.2) \) - PSO convergence behavior. It controls the impact of the previous history of velocities on the current one.
- \( c_1, c_2 \) - proper fine-tuning may result in faster convergence and alleviation of the local minima (\( c_1 + c_2 \leq 4 \)).
- \( r1, r2 \) - used to maintain the diversity of the population, and they are uniformly distributed in the range [0, 1].
The general flowchart of Classical PSO is illustrated as follows:

\[
W_i = W_{max} - \left(\frac{W_{max} - W_{min}}{iter_{max}}\right)iter \quad (xv)
\]

Where, \( W_{max} \) is the initial weight, \( W_{min} \) is the final weight, \( iter_{max} \) is the maximum iteration number, \( iter \) is the current iteration number.

The equation \((xiii)\) is used to calculate the particle's new velocity according to its previous velocity and the distances of its current position from its own best experience (position) and the group's best experience. Then the particle flies towards a new position according to equation \((xiv)\). The performance of each particle is measured according to a predefined fitness function, which is related to the problem to be solved.

V. A BASIC PSO ALGORITHM

The step by step procedure of PSO algorithm is given as follows:

- Initialize a population of particles as

\[
P_i = (P_{i1}, P_{i2}, P_{i3}... P_{iN}) \quad (xvi)
\]

- ‘N’ is number of generating units. Population is initialized with random values and velocities within the d-dimensional search space.

- Initialize the maximum allowable velocity magnitude of any particle \( V_{max} \).

\quad \text{Evaluate the fitness of each particle and assign the particle’s position to P-best position and fitness to P-best fitness. Identify the best among the P-best as G-best and store the fitness value of G-best.}

\quad \text{Change the velocity and position of the particle according to equations (xiii) and (xiv), respectively.}

\quad \text{For each particle, evaluate the fitness, if all decisions variable are within the search ranges.}

\quad \text{Compare the particle's fitness evaluation with its previous P-best. If the current value is better than the previous P-best, then set the P-best value equal to the current value and the P-best location equal to the current location in the d-dimensional search space.}

\quad \text{Compare the best current fitness evaluation with the population G-best. If the current value is better than the population G-best, then reset the G-best to the current best position and the fitness value to current fitness value.}

\quad \text{Repeat steps 2-5 until a stopping criterion, such as sufficiently good G-best fitness or a maximum number of iterations/function evaluations is met.}

VI. IMPLEMENTATION OF CLASSICAL PSO FOR ELD SOLUTION

The main objective of ELD is to obtain the amount of real power to be generated by each committed generator, while achieving a minimum generation cost within the constraints. The details of the implementation of PSO components are summarized in the following subsections.

The search procedure for calculating the optimal generation quantity of each unit is summarized as follows:

- Initialization of the swarm: For a population size \( P \), the particles are randomly generated in the range 0-1 and located between the maximum and the minimum operating limits of the generators. If there are \( N \) generating units, the \( i \)th particle is represented as

\[
P_i = (P_{i1}, P_{i2}, P_{i3}... P_{iN}) \quad (xvii)
\]

The \( j \)th dimension of the \( i \)th particle is allocated a value \( P_{ij} \) as given below to satisfy the constraints:

\[
P_{ij} = P_{j\min} + r (P_{j\max} - P_{j\min}) \quad (xviii)
\]

Here \( r [0, 1] \)

- Defining the evaluation function: The merit of each individual particle in the swarm is found using a fitness function called evaluation function. The popular penalty function method employs functions to reduce the fitness of the particle in proportion to the magnitude of the equality constraint violation (xii).

The evaluation function is defined to minimize the non-smooth cost function given by equation (ii).The evaluation function is given as

\[
\text{Min}(x)=I(x)+
\]

- Initialization of P-best and G-best: The fitness values obtained above for the initial particles of the swarm are set as the initial Pbest values of the particle.

The best value among all the Pbest values is identified as G-Best.

- Evaluation of velocity: The update in velocity is done by equation (xiii).

Check the velocity constraints of the members of each individual from the following conditions [26 - 29]: If, \( V_{id}(k+1) > V_{dmax} \), then \( V_{id}(k+1) = V_{dmax} \), \( V_{id}(k+1) < V_{dmin} \) then, \( V_{id}(k+1) = V_{dmin} \) \quad (xix)

Where, \( V_{dmin} = -0.5 P_{gmin} \), \( V_{dmax} = +0.5 P_{gmax} \)
• Modify the member position of each individual
• \( P_g \) [27 - 30] according to the equation
\[
P_{gid}(k+1) = P_{gid}(i) + V_{id}(k+1)
\] (xx)
\( P_{gid}(k+1) \) must satisfy the constraints, namely the generating limits. If \( P_{gid}(k+1) \) violates the constraints, then \( P_{gid}(k+1) \) must be modified towards the nearest margin of the feasible solution.

> If the evaluation value of each individual is better than previous P-best, the current value is set to be P-best. If the best P-best is better than G-best, the best P-best is set to be G-best. The corresponding value of fitness function is saved.

> If the number of iterations reaches the maximum, then go to step 10. Otherwise, go to step 2.

• The individual that generates the latest G-best is the optimal generation power of each optimal generation power of each unit with the minimum total generation cost.

The flowchart of implementation of PSO for ELD:

Figure 2: The flowchart of implementation of PSO

Experimental Parameters
• Initial Population
The initial populations are generated randomly, and it is a set of \( n \) particles at time \( t \).
• Swarm
It is an apparently disorganized population of moving particles that tend to cluster together while each particle seems to be moving in a random direction.
• Population Size
From the earlier research performed by Eberhart and Shi, it is proved that the performance of the standard algorithm is not sensitive to the population size but to the convergence rate.
• Search Space
The range in which the algorithm computes the optimal control variables is called search space. The algorithm will search for the optimal solution in the search space between 0 and 1. When any of the optimal control values of any particle exceed the searching space, the value will be reinitialized.
• Time-Varying Inertia Weight (TVIW)
In order to improve the performance of the PSO, the time-varying inertia weight was proposed in [7]. This inertia weight linearly decreases with respect to time. Generally for initial stages of the search process, large inertia weight to enhance the global exploration (searching new area) is recommended while, for last stages, the inertia weight is reduced for local exploration (fine tuning the current search area). The mathematical expression for TVIW:
\[
w = (w_1 - w_2) \left( \frac{\text{iter}_{\max} - \text{iter}}{\text{iter}_{\max}} \right) + w_2
\]
where \( w_1 \) is initial value of the inertia weight, \( w_2 \) final value of the inertia weight, \( \text{iter} \) current iteration, \( \text{iter}_{\max} \) maximum number of allowable iterations.

• Inertia Weight Used in the Present Work (GLbestIW)
The GLbestIW method is proposed in [28] in which, the inertia weight is neither set to a constant value nor set as linearly decreasing time-varying function. The inertia weight is defined as a function of local best (pbest) and global best (gbest) values of the particles in each generation. The GLbest inertia weight is given by the following equation
\[
w_{gl} = \left( 1.1 - \frac{g_{best}}{p_{best}} \right)
\]

VIII. RESULTS
Case-1 3-unit system [29]
The system contains 3 thermal units, Data as follows:
\[
\begin{align*}
F_1 &= 0.00524P_1^2 + 8.66 P_1 + 224.489 \text{ KES/Hr} \\
F_2 &= 0.00608P_2^2 + 10.05 P_2 + 93.676 \text{ KES/Hr} \\
F_3 &= 0.00592P_3^2 + 9.75 P_3 + 40.469 \text{ KES/Hr}
\end{align*}
\]
240 MW \( \leq P_1 \leq 90 \text{ MW} \)
238 MW \( \leq P_2 \leq 85 \text{ MW} \)
100 MW \( \leq P_3 \leq 20 \text{ MW} \)

B = Coefficient Matrix:
\[
B = [0.000134 \quad 0.0000176 \quad 0.000183 \\
0.000176 \quad 0.000153 \quad 0.000282 \\
0.000183 \quad 0.000282 \quad 0.00162]
\]

VII. EXPERIMENT
Determination of whether neighborhood topology could affect the convergence. The ring topology is also known as the best version in PSO where fitness dilutes proportionally with respect to the distance of its k immediate neighbors of the population.

Figure 3: Ring and star topologies
The corresponding loads are given as 300 MW and 450 MW respectively.

- Table-1 Three Generator system with optimal scheduling without losses by PSO

<table>
<thead>
<tr>
<th>Load Demand (MW)</th>
<th>Pg1 (MW)</th>
<th>Pg2 (MW)</th>
<th>Pg3 (MW)</th>
<th>Fuel cost (KES/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>300 MW</td>
<td>161.076541</td>
<td>317.49659</td>
<td>155.573291</td>
<td>4049.022</td>
</tr>
<tr>
<td>450 MW</td>
<td>152.768436</td>
<td>182.873239</td>
<td>39.238932</td>
<td>6439.167</td>
</tr>
</tbody>
</table>

(i) Simulation Results of 3 Unit without Loss with 450 MW Load

![Graph between G-best solutions and Cost in KES/hr for a load of 450 mw](image)

Evaluuated results obtained from PSO method with conventional method and their comparison is shown in the tables below:

- Table- 2 Comparison of different methods without losses of 3-unit system

<table>
<thead>
<tr>
<th>Power Demand (MW)</th>
<th>Fuel Cost (KES/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Conventional Method</td>
</tr>
<tr>
<td>300</td>
<td>4049.05</td>
</tr>
<tr>
<td>450</td>
<td>6439.20</td>
</tr>
</tbody>
</table>

Above table shows that PSO method provides better results.

<table>
<thead>
<tr>
<th>Load Demand (MW)</th>
<th>Pg1 (MW)</th>
<th>Pg2 (MW)</th>
<th>Pg3 (MW)</th>
<th>Fuel cost (KES/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>300 MW</td>
<td>120.458656</td>
<td>87.893538</td>
<td>23.134332</td>
<td>4115.513</td>
</tr>
<tr>
<td>450 MW</td>
<td>161.957099</td>
<td>175.374458</td>
<td>66.783262</td>
<td>6213.839</td>
</tr>
</tbody>
</table>

- Table-3 Three Generator system with optimal scheduling with losses by PSO

Transmission losses which can be calculated with the help of loss matrix $B_{mn}$ provided in section.

(ii) Simulation Results of 3 Unit with Loss with 450 MW Load

![Graph between G-best solutions and Cost in KES/hr for a load of 450 mw](image)

Table- 4 Comparison of different methods including losses of 3-unit system

<table>
<thead>
<tr>
<th>Power Demand (MW)</th>
<th>Fuel Cost (KES/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Conventional Method</td>
</tr>
<tr>
<td>300</td>
<td>4116.86</td>
</tr>
<tr>
<td>450</td>
<td>6215.14</td>
</tr>
</tbody>
</table>

The new concept of defining the inertia weight in terms of the personal and global best values helps the PSO to perform better in solving any high-dimensional optimal control problem with faster convergence and accuracy. The impacts of inertia weight variants are analyzed.

- Table- 5 Comparison of different PSO methods

<table>
<thead>
<tr>
<th>S/no.</th>
<th>Number of trials</th>
<th>Method</th>
<th>Min. cost</th>
<th>Max. cost</th>
<th>Average cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>CIW</td>
<td>802.959</td>
<td>822.351</td>
<td>809.587</td>
</tr>
<tr>
<td></td>
<td></td>
<td>TVIW</td>
<td>802.741</td>
<td>824.391</td>
<td>809.741</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GLBestIW</td>
<td>801.113</td>
<td>816.277</td>
<td>807.828</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>CIW</td>
<td>802.843</td>
<td>804.921</td>
<td>803.881</td>
</tr>
<tr>
<td></td>
<td></td>
<td>TVIW</td>
<td>802.543</td>
<td>802.551</td>
<td>802.494</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GLBestIW</td>
<td>801.843</td>
<td>801.845</td>
<td>801.844</td>
</tr>
<tr>
<td>3</td>
<td>200</td>
<td>CIW</td>
<td>802.843</td>
<td>804.913</td>
<td>802.843</td>
</tr>
<tr>
<td></td>
<td></td>
<td>TVIW</td>
<td>802.543</td>
<td>802.852</td>
<td>802.543</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GLBestIW</td>
<td>801.843</td>
<td>801.845</td>
<td>801.843</td>
</tr>
</tbody>
</table>

A new inertia weight is proposed in terms of the global best and personal best values of the objective function. The results here hence indicate that the improved performance of the PSO can be obtained by carefully selecting the inertia weight.

IX. CONCLUSION

We can draw important conclusions on the basis of the work done. Some important conclusions are given below

**Three Unit Systems:**

In PSO method selection of parameters $c_1$, $c_2$ and $W$ is very much important. It is stated in various research papers that the good results are obtained when $c_1 = 2.0$ and $c_2 = 2.0$ and $W$ value is varied from 0.9 to 0.4 for both cases loss neglected and loss included. It is evident that Classical PSO gives better result than conventional methods.
In PSO method numbers of iterations are not much affected when the transmission line losses are considered. In both cases for loss included and loss neglected it is approximately 50 iterations for Classical PSO method.

To verify the feasibility of the proposed PSO method, three different power systems were tested, under the same evaluation function and individual definition. 3 sets of trials were performed to observe the evolutionary process and to compare their solution quality, convergence characteristic, and computation efficiency. From the experiences of many experiments the following parameters are selected for the particle swarm optimization algorithm to solve the above test cases and are tabulated. For implementing the above algorithm, the simulation studies were carried out on P-IV, 2.4 GHz, 512 MB DDR RAM system in MATLAB environment.

Six Unit Systems:
The selection of parameters is same as $c1=2.0, c2=2.0, W$ is varying from 0.9 to 0.4. It was evident that Classical PSO method gives better result than the conventional method as the cost is reduced. Table-5 gives the minimum, maximum, and average costs for 1st trial, 100 trials and 200 trials for all the three PSO methods under consideration. It can be seen that the minimum cost as well as the average cost produced by GLBestIW PSO is the least as compared to other methods. This emphasizes the better quality solution of the proposed method.

It is shown through different trials that the GLBestIW PSO outperforms other methods in terms of high quality solution, consistency, faster convergence, and accuracy. Overall we can conclude that today when there is competition amongst power generating companies, fast emerging difference between demand and supply then we need to develop a requisite for proper operation policies for power generating companies. It can be accomplished only when a proper mathematical formulation of ELD problem is there and all practical constraints are taken into account. PSO has paid a lot of attention for solution of such problems, as it does not suffer from sticking into local optimal solution, dependability on initial variables and curse of dimensionality.

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REFERENCES


